Patterns and Rules

Patterns

EXERCISE 11A

1. Complete the following number patterns.

(a) 2, 4, 6, 8, __, __, __.
(b) 20, 17, 14, 11, __, __, __.
(c) 1, 3, 9, 27, __, __, __.
(d) 0, 4, 8, 12, __, __, __.
(e) 100, 90, 80, 70, __, __, __.
(f) 64, 32, 16, 8, __, __, __.
(g) 3, 6, 12, 24, __, __, __.
(h) 100 000, 10 000, 1000, __, __, __.
(i) 0.2, 2, 20, 200, __, __, __.
(j) 5, 10, 15, 20, __, __, __.

2. Explain how to find a number in each of the number patterns in question 1.

Examples

1. 1, 3, 5, 7, __, __, __.
In this pattern a number is found by **adding 2** to the number before it.
Answer: **+2**

2. 1, 4, 16, 64, __, __, __.
In this pattern a number is found by **multiplying by 4** the number before it.
Answer: **×4**

3. Find the first five numbers in the number patterns that have the following starting numbers and method of finding the other numbers.

Examples

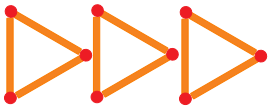
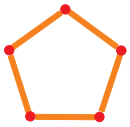
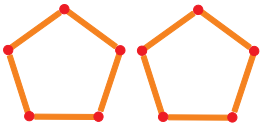
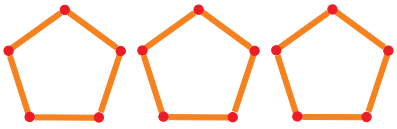
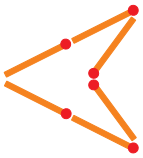
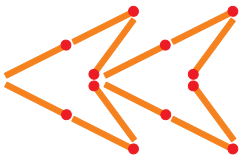
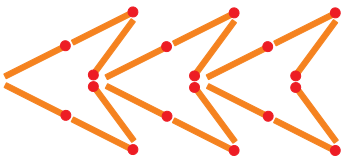
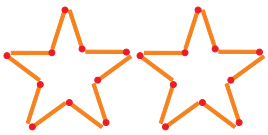
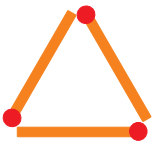
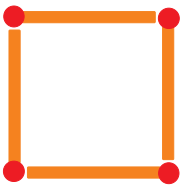
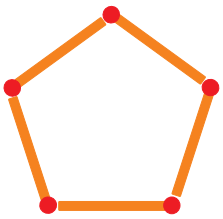
1. 3 (+4)
Starting number = 3
Each number is found by **adding 4** to the number before it.
First five numbers are:
3, 7, 11, 15, 19

2. 2 (×3)
Starting number = 2
Each number is found by **multiplying by 3** the number before it.
First five numbers are:
2, 6, 18, 54, 162

(a) 7 (+3) (b) 50 (-5) (c) 1 (×10) (d) 10 (+10) (e) 2187 (÷3)

4. Find the eighth number in each of the patterns in question 3.

5. The table below shows several patterns that can be made with matches.

	Step 1	*Step 2*	*Step 3*
A			
B			
C			
D			
E			

(a) Describe how the step number can be used to find the number of matches needed for each step of the patterns.

Example In pattern A the number of matches can be found by **multiplying the step number by 3**.

(b) Find the number of matches needed to form **step 8** of each pattern.

Example For pattern A at step 8 there would need to be:
$8 \times 3 = 24$ matches

Symbols and Rules

EXERCISE 11B

1. Refer to the patterns in question 5 Exercise 11A.
Let the symbol m represent the number of matches and s represent the step number.
Use these symbols to find a rule for each pattern that can be used to find the number of matches needed for a step number.

Example In pattern A the number of matches, m, is equal to 3 times the step number, s.

$$m = 3 \times s$$

2. (a) For the pattern below, find how many matches are needed to form: (i) step 1 (ii) step 2 (iii) step 3 (iv) step 4 (v) step 6

(b) Copy and complete this table showing the number of matches, m, needed for the different steps, s, of the pattern.

s	m
1	
2	
3	
4	
5	
6	

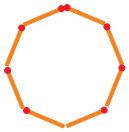
(c) How many matches would be needed to form step 20 of the pattern?

(d) What step of the pattern would need:
(i) 60 matches?
(ii) 72 matches?

(e) Copy and complete the following rule that can be used to find the number of matches, m, for a step number, s.

$$m = \_\_ \times s$$

3. (a) For the pattern below, copy and complete the table showing the number of matches, m, and the step number, s.



Step 1

Step 2

Step 3

Step 4

s	m
1	
2	
3	
4	
5	
6	

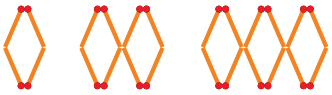
(b) Copy and complete the following rule:

$m = s +$ __

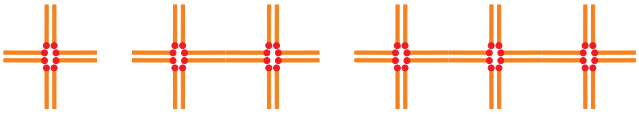
4. For each of the patterns below:
(i) Copy and complete the table showing the number of matches, m, and the step number, s.
(ii) Write a rule that could be used to find the number of matches for a step number.

$m =$

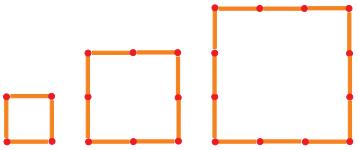
s	m
1	
2	
3	
4	
5	
6	

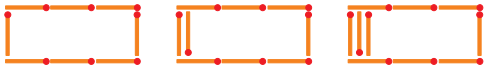
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

5. Find the rules for the symbols shown in the following tables.

Examples

s	m
1	2
2	4
3	6
4	8
5	10
6	12

$m = 2 \times s$

a	b
1	3
2	4
3	5
4	6
5	7
6	8

$b = a + 2$

M	N
11	10
12	11
13	12
14	13
15	14
16	15

$N = M - 1$

(a)

s	m
1	5
2	10
3	15
4	20
5	25
6	30

$m =$

(b)

A	B
1	5
2	6
3	7
4	8
5	9
6	10

$B =$

(c)

p	q
1	0
2	1
3	2
4	3
5	4
6	5

$q =$

(d)

c	t
1	10
2	20
3	30
4	40
5	50
6	60

$t =$

(e)

x	y
1	21
2	22
3	23
4	24
5	25
6	26

$y =$

(f)

F	T
20	10
18	9
16	8
14	7
12	6
10	5

$T =$

(g)

W	T
15	9
14	8
13	7
12	6
11	5
10	4

$T =$

(h)

h	k
50	10
40	8
30	6
20	4
10	2
0	0

$k =$

(i)

α	β
0	8
6	14
11	19
20	28
34	42
55	63

$\beta =$

6. For each of the following rules, copy and complete the table shown.

(a) $n = m + 5$ (b) $n = 11 \times m$ (c) $n = m + 15$
(d) $n = m \div 10$ (e) $n = m - 6$ (f) $n = m \div 2$
(g) n is found by multiplying m by 2.
(h) n is found by adding 10 to m.
(i) n is found by subtracting 1 from m.

m	n
10	
20	
30	
40	
50	
60	

7. Use the rule shown to answer the following questions.

$y = x + 6$

(a) Find y if: (i) $x = 6$ (ii) $x = 10$ (iii) $x = 22$ (iv) $x = 38$
(b) Find x if: (i) $y = 7$ (ii) $y = 10$ (iii) $y = 26$ (iv) $y = 43$

8. For each of the following rules, copy and complete the table shown.

(a) $y = x + 5$ (b) $y = 3 \times x$ (c) $y = x + 15$
(d) $y = x \div 3$ (e) $y = x - 2$ (f) $y = 2 \times x$
(g) y is found by multiplying x by 10.
(h) y is found by adding 10 to x.
(i) y is found by subtracting 1 from x.

x	y
3	
6	
	30
12	
21	
	60

9. A sweets shop was selling bags of chocolates. There were 10 chocolates in each bag.

(a) How many chocolates would there be in:
(i) 2 bags? (ii) 5 bags?
(iii) 10 bags? (iv) 20 bags?

(b) Which of the following rules could be used to find the number of chocolates, c, in a certain number of bags, b?

A $c = 10 \times b$ **B** $c = b + 10$ **C** $c = b - 10$ **D** $c = b \div 10$

(c) How many bags would be needed for:
(i) 30 chocolates? (ii) 70 chocolates? (iii) 500 chocolates?

(d) Which of the following rules could be used to find the number of bags, b, needed for a certain number of chocolates, c?

A $b = 10 \times c$ **B** $b = c + 10$ **C** $b = c - 10$ **D** $b = c \div 10$

10. Alisha is 6 years older than George.

(a) How old will Alisha be when George is:

(i) 10 years old? (ii) 18 years old? (iii) 39 years old?

(b) Which of the following rules could be used to find Alisha's age, A, given George's age, G?

A $A = 6 \times G$ **B** $A = G + 6$ **C** $A = G - 6$ **D** $A = G \div 6$

(c) How old will George be when Alisha is:

(i) 26 years old? (ii) 38 years old? (iii) 53 years old?

(d) Which of the following rules could be used to find George's age, G, given Alisha's age, A?

A $G = 6 \times A$ **B** $G = A + 6$ **C** $G = A - 6$ **D** $G = A \div 6$

11. Glenda can walk 4 kilometres every hour when bush-walking.

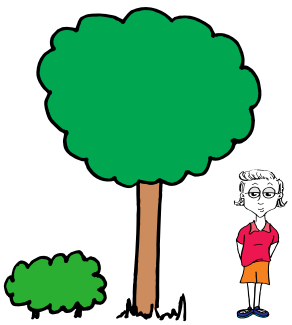
(a) How far would she walk in:

(i) 3 hours? (ii) 7 hours?

(b) Which of the following rules could be used to find the distance, d, Glenda would walk in h hours.

A $d = 4 \times h$ **B** $d = h + 4$

C $d = h - 4$ **D** $d = h \div 4$



(c) How many hours would it take Glenda to walk:

(i) 16 km? (ii) 24 km?

(d) Which of the following rules could be used to find how many hours, h, it would take Glenda to walk d kilometres?

A $h = 4 \times d$ **B** $h = d + 4$ **C** $h = d - 4$ **D** $h = d \div 4$

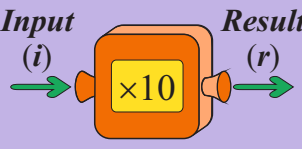
Number Machines

EXERCISE 11C

1. Copy and complete the tables for each of the number machines below.

In a number machine a number that is put into the machine is called the ***input*** (***i***).
This number is processed and the number that comes out of the machine is called the ***result (r)***.
The operation stated on the machine is how the number is processed.
In the number machine below the process is ×10. This means that each input number is multiplied by 10 to find the result.

Example

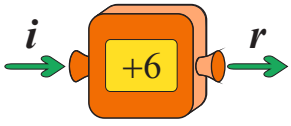


i	1	3	12	20
r				

Answer

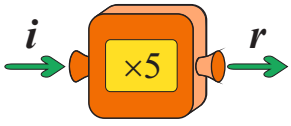
i	1	3	12	20
r	**10**	**30**	**120**	**200**

(a)



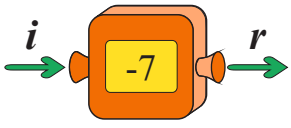
i	1	4	9	15
r				

(b)



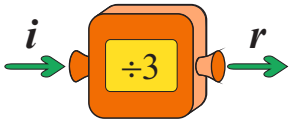
i	2	3	9	11
r				

(c)



i	8	13	19	26
r				

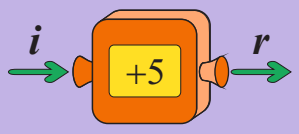
(d)



i	3	9	21	36
r				

2. Copy and complete the tables for each of the number machines below.

Example



i				
r	8	11	17	22

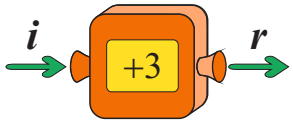
To find the input knowing the result you need to work 'backwards'.

$i + 5 = r$
For $r = 8$
$i + 5 = 8$
$i = 8 - 5$
$i = 3$

Answer

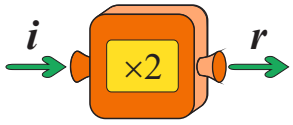
i	**3**	**6**	**12**	**17**
r	8	11	17	22

(a)



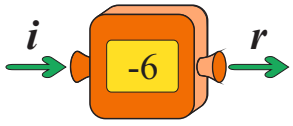
i				
r	5	9	13	19

(b)



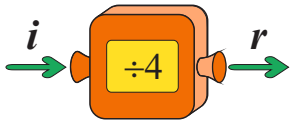
i				
r	6	8	14	20

(c)



i				
r	2	5	9	13

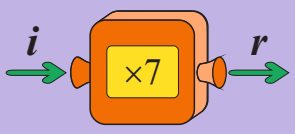
(d)



i				
r	2	5	8	11

3. Copy and complete the tables for each of the number machines below.

Example



i	2	6		
r			56	70

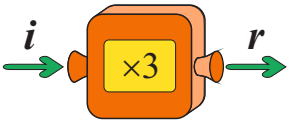
Answer

i	2	6	**8**	**10**
r	**14**	**42**	56	70

(a)

i	3	5		
r			13	17

(b)

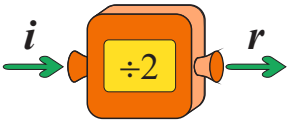


i	2	4		
r			18	24

(c)

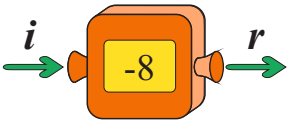
i	8		15	
r		6		17

(d)



i	6			22
r		4	7	

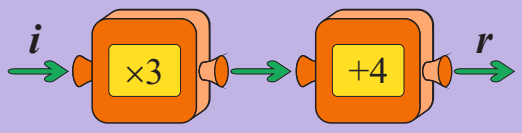
(e)



i		13		26
r	0		9	

4. Copy and complete the tables for the number machines below.

Example



i	2	4	6	9
r				

In these problems there are two steps. The input is processed by the first number machine then this result is processed by the second number machine to give the final result.

For $i = 2$

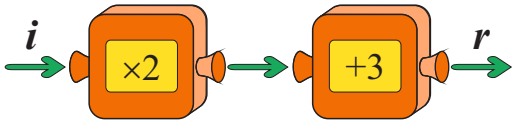
The input is processed by the first number machine: $2 \times 3 = 6$

This result is then processed by the second number machine: $6 + 4 = 10$

Answer

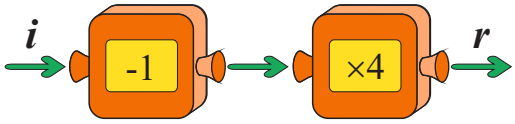
i	2	4	6	9
r	**10**	**16**	**22**	**31**

(a)



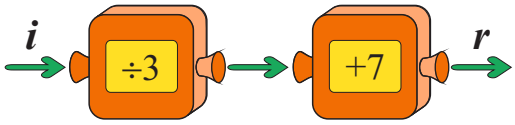
i	1	3	7	9
r				

(b)



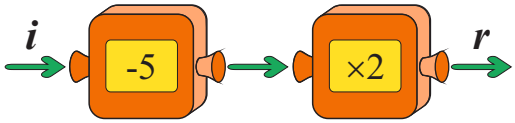
i	3	5	6	10
r				

(c)



i	6	9	15	21
r				

(d)

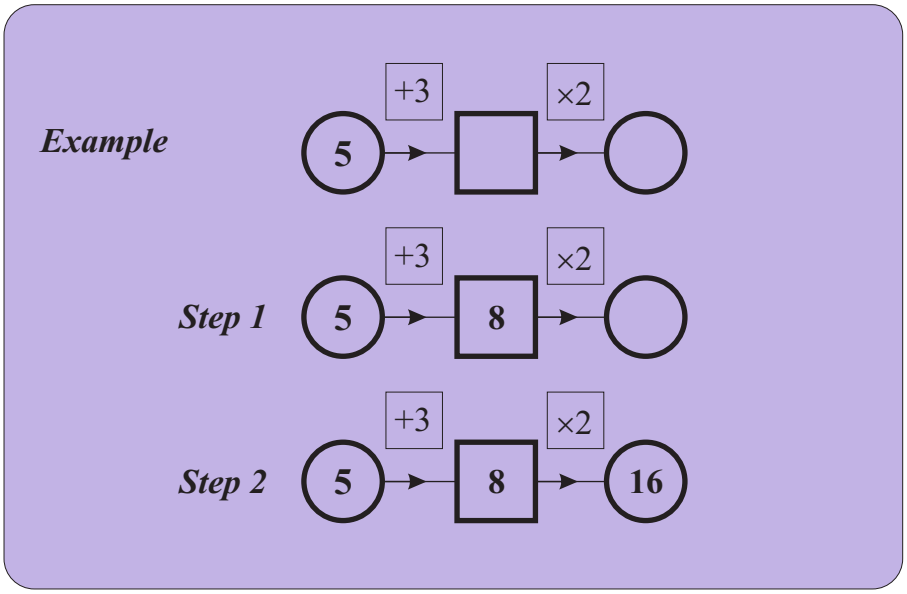


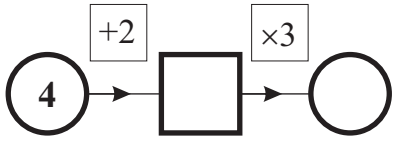
i	8	10	13	17
r				

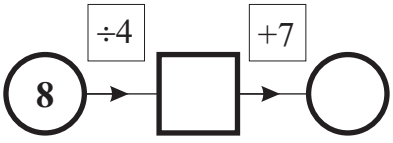
Flowcharts and Rules

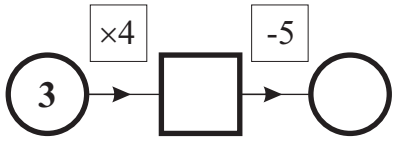
EXERCISE 11D

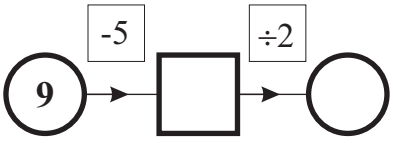
1. Complete the following flowcharts.

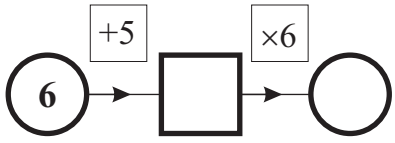


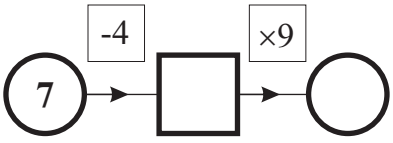
(a) 

(b) 

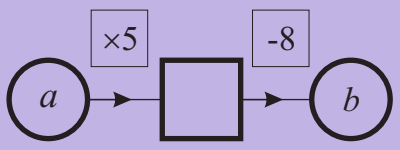
(c) 

(d) 

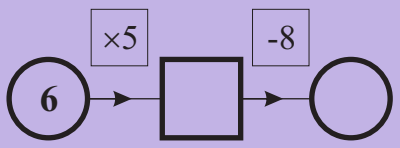
(e) 

(f) 

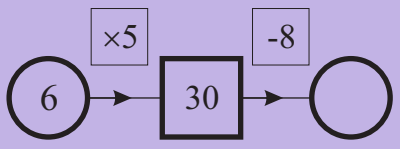
Example For the following flowchart find b when $a = 6$.



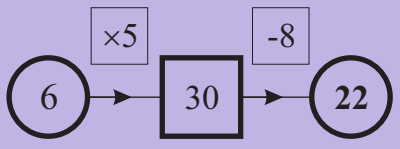
Step 1
Let $a = 6$



Step 2

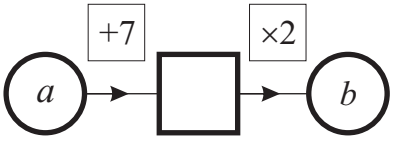


Step 3

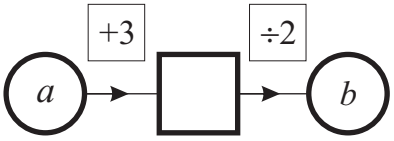


Answer: b = 22

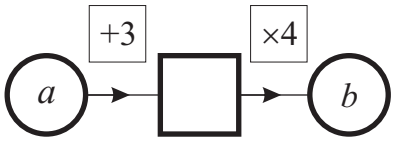
2. For the following flowchart find b when $a = 3$.



3. For the flowchart shown below find b for the following values of a.
(a) $a = 5$ (b) $a = 7$ (c) $a = 11$ (d) $a = 27$



4. For the following flowchart copy and complete the table by finding the values of b for the values of a shown.



a	b
1	
2	
3	
4	

5. Match the tables of values below with their correct flowcharts. Check that the flowchart works for all pairs of numbers in the table.

(a)

a	b
1	6
2	10
3	14
4	18

(b)

a	b
1	6
2	8
3	10
4	12

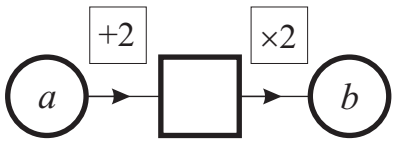
(c)

a	b
1	6
2	13
3	20
4	27

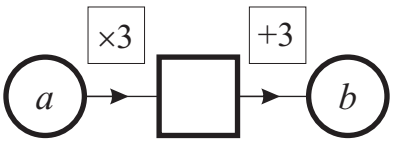
(d)

a	b
1	6
2	9
3	12
4	15

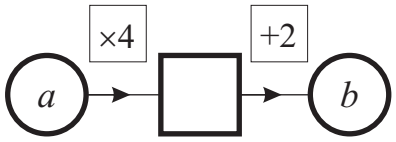
A



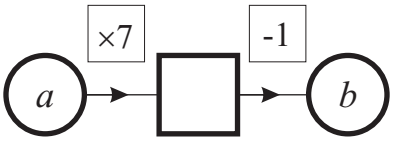
B



C

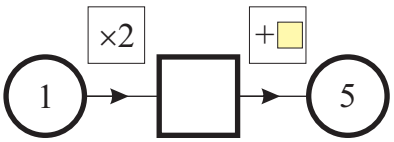


D

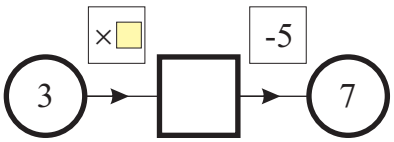


6. Complete the following flowcharts by finding the missing numbers.

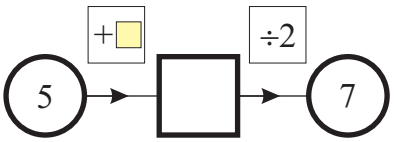
(a)



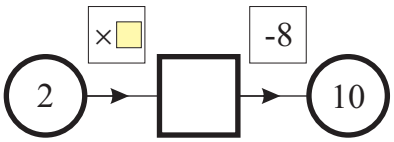
(b)



(c)

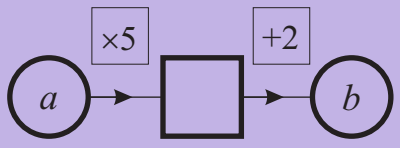


(d)



In algebra, mathematical rules are called ***equations***.
Write the following flowchart as an equation.

Example 1



The value of b is found by multiplying a by 5 then adding 2 to the result.
This can be written as a mathematical equation:

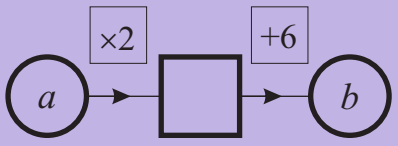
$$b = a \times 5 + 2$$

This is written as: $b = 5 \times a + 2$

In algebra it is not necessary to show the multiplication sign (×) and the number is always written before the symbol when multiplying.
The above equation would be written as shown below.

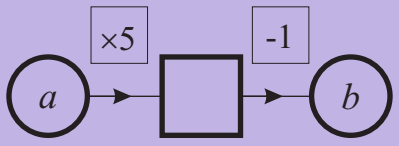
Answer: $\boldsymbol{b = 5a + 2}$

Example 2



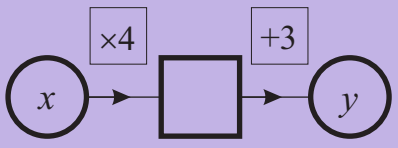
Answer: $\boldsymbol{b = 2a + 6}$

Example 3



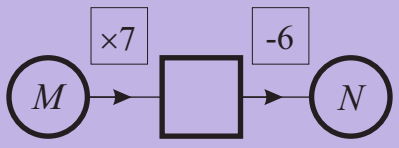
Answer: $\boldsymbol{b = 5a - 1}$

Example 4



Answer: $\boldsymbol{y = 4x + 3}$

Example 5

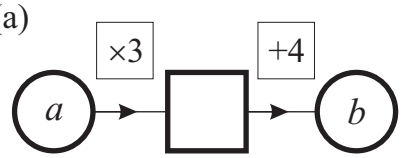


Answer: $\boldsymbol{N = 7M - 6}$

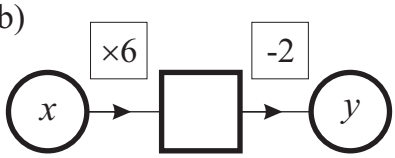
EXERCISE 11E

1. Write the following flowcharts as equations.

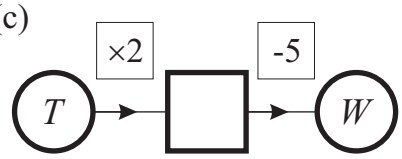
(a)



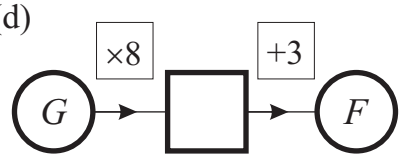
(b)



(c)

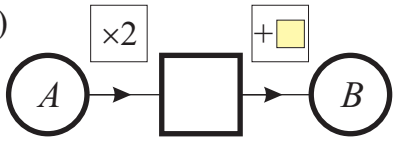


(d)

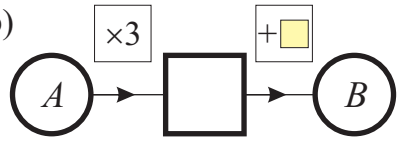


2. Copy and complete the following flowcharts for $A = 2$ and $B = 9$.

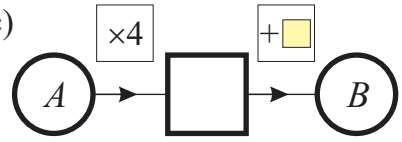
(a)



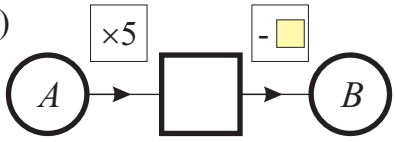
(b)



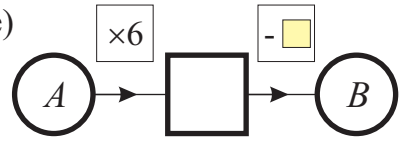
(c)



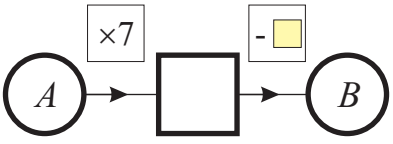
(d)



(e)



(f)



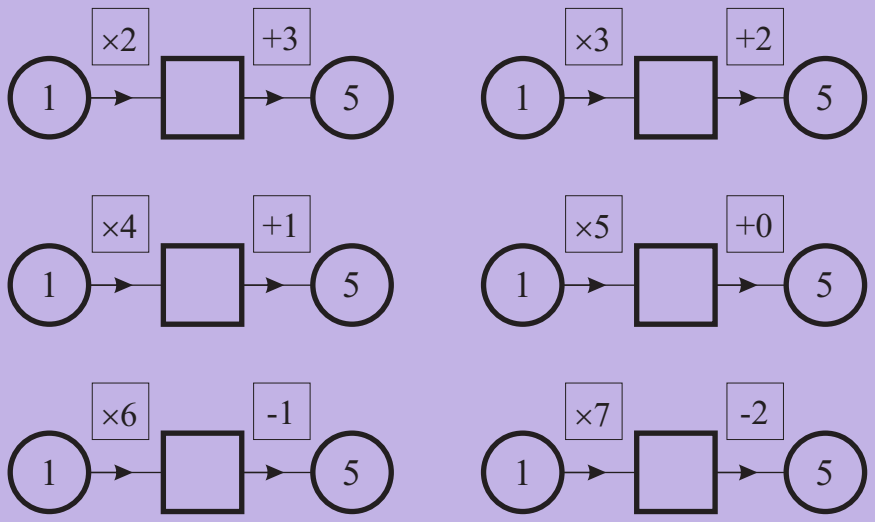
3. Write the equations for each of the flowcharts in question 2.

Example

Find the flowchart and equation for the table of values shown.

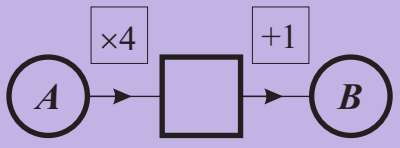
A	*B*
1	5
2	9
3	13
4	17

Step 1 Using the first pair of numbers the flowchart could be one of many. Several of these are shown below.



Step 2 By trial and error find which of these flowcharts works for all the other pairs of numbers.

Answer:

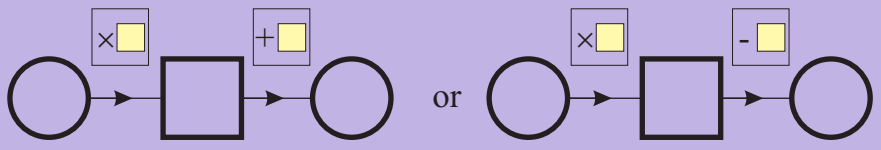


Equation: $B = 4A + 1$

Always check that the equation works for all the pairs of numbers.

4. For the following tables of values:
(i) draw the flowchart.
(ii) find the equation.

Note: the flowcharts will be one of the following.



(a)

a	b
1	3
2	5
3	7
4	9

(b)

c	d
1	2
2	5
3	8
4	11

(c)

T	S
1	5
2	7
3	9
4	11

(d)

v	w
1	5
2	8
3	11
4	14

(e)

G	H
1	5
2	9
3	13
4	17

(f)

j	k
1	1
2	5
3	9
4	13

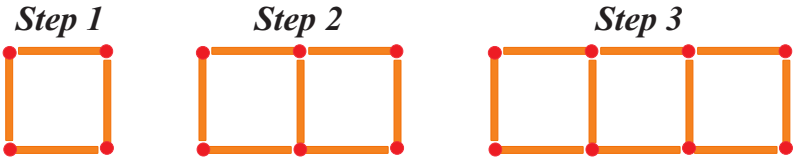
(g)

m	n
1	7
2	9
3	11
4	13

(h)

P	Q
1	0
2	3
3	6
4	9

5. (a) Copy and complete this table showing the number of matches, m, needed for each step number, s.



s	m
1	4
2	
3	
4	

(b) Draw the flowchart that could be used to find the number of matches (m) for a certain step number (s).
(c) Find the equation for this flowchart.
(d) How many matches would be needed for: (i) step 6? (ii) step 10?

6. For each of the following patterns:

(i) copy and complete the table showing the number of matches, m, needed for each step number, s.

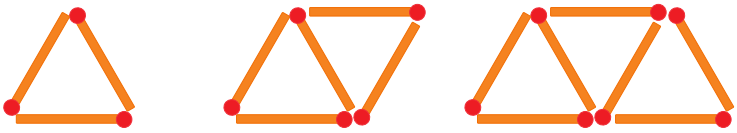
(ii) draw the flowchart.

(iii) write the equation.

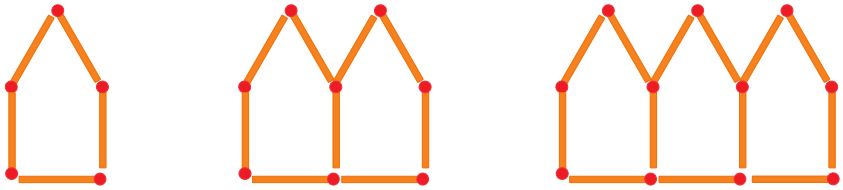
(iv) find the number of matches needed for step 10.

s	m
1	
2	
3	
4	

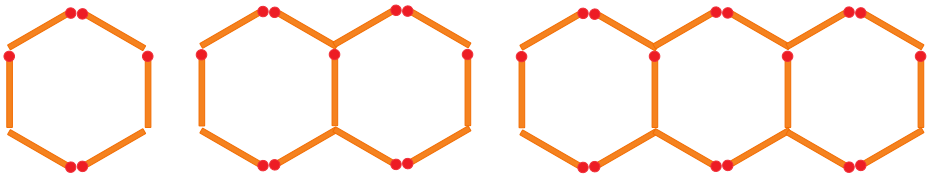
(a)



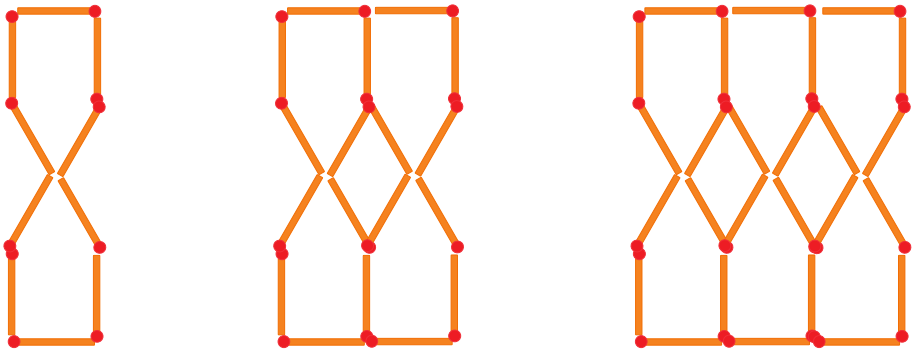
(b)



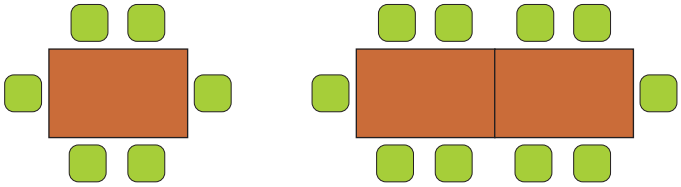
(c)



(d)



7. Jane and Hamish are getting married. They are organising the tables for the reception and can join any number of tables, end to end, as shown below. The number of chairs around each set of tables is shown.
Six chairs can be placed around one table, 10 chairs around two tables, etc.



t	c
1	6
2	10
3	
4	

(a) Copy and complete this table showing the number of chairs, c, that could be placed around t tables.

(b) Find a rule that could be used to find the number of chairs that could be placed around a given number of tables.

(c) How many chairs could be placed around 6 joined tables?

There are 50 guests coming to the wedding. Jane and Hamish have decided to have the tables arranged so there are no more than four tables joined together.

(d) Copy and complete the table below showing all the possible ways that the tables could be arranged to seat exactly 50 guests. Show the number of groups of 1, 2, 3 or 4 tables.
There are ***eight*** different combinations.

Example

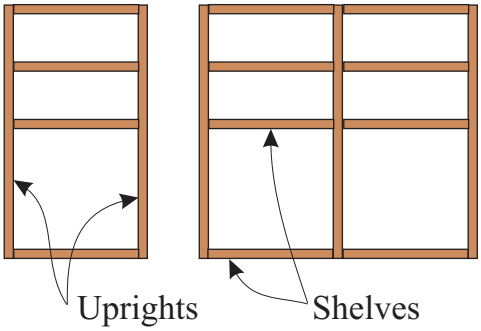
6 single tables + 1 group of 3 tables
$= (6 \times 6) + (1 \times 14)$
$= 36 + 14$
$= 50$

Number of Groups	Number of Tables in each Group			
	1	2	3	4
	6		1	

(e) Which combinations require the least number of tables?

PROBLEM SOLVING

1. Roland is a cabinet maker who builds bookcases as shown here.
The bookcases are made using four shelves fixed in between uprights.
Roland can make the bookcases any size by adding more uprights and shelves.



(a) Copy and complete this table showing the number of shelves, s, that would be needed for a given number of uprights u.

u	s
2	4
3	8
4	
5	

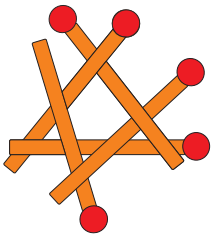
(b) Write a rule that could be used to find the number of shelves, s, needed for u uprights.
(c) How many shelves would be needed if there are 8 uprights?
(d) How many uprights would be needed if the bookcase is to have 20 shelves?
(e) Roland has orders for 8 bookcases - two with 8 shelves, three with 12 shelves and three with 16 shelves. How many uprights will he need to cut to make these bookcases?

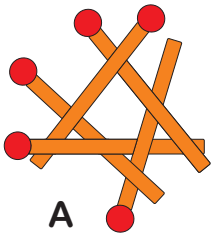
2. A greengrocer bought a full box of avocados.
A customer bought half the avocados in the box.
Another customer bought half of the remaining avocados.
Another customer bought three avocados.
Another customer bought the remaining two avocados.
How many avocados were in the full box?

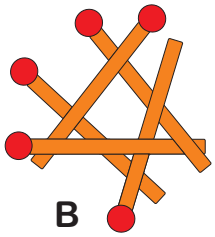
3. Iona is five years younger than her brother Byron.
Karen is their mother and is three times Iona's age.
Marlene is Iona's grandmother and is five times Iona's age.
The sum of all their ages is 125.
How old is Iona?

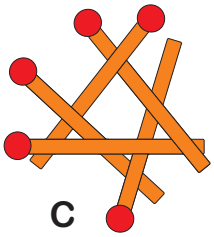
PUZZLES

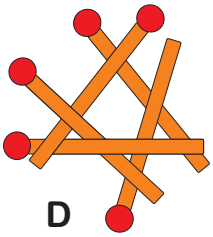
1. Five matches are dropped onto a glass table. This diagram shows how they landed. Which of the diagrams below show what the matches would look like if viewed from under the glass table?









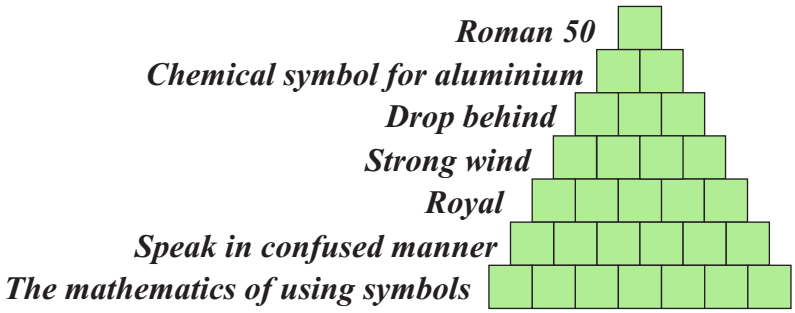


2. All the words that are needed to complete this puzzle can be found in this chapter. When you have found all the words the letters in boxes spell the answer to the following riddle.

		A			I		G	
M		C			N			
					U		B	R
	Y	M			L			
						E	P	
			P			T	E	
	F		O		C		R	

What did the prehistoric carpenter call one of his tools?

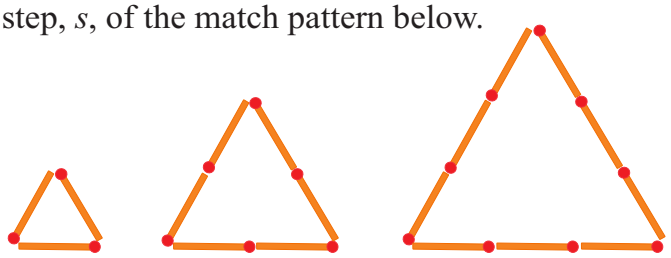
3. Complete this diagram below by finding the word on each line. Each word can be found by adding one letter to the line above it and rearranging the letters. A clue for each word is given.



CHAPTER REVIEW

1. Complete the following number patterns.
(a) 1, 4, 7, 10, __, __, __. (b) 10, 20, 40, 80, __, __, __.

2. (a) Copy and complete the table showing the number of matches, m, needed for each step, s, of the match pattern below.



Step 1 *Step 2* *Step 3*

s	m
1	
2	
3	
4	
5	
6	

(b) Draw the flowchart.
(c) Write the rule.
(d) How many matches would be needed for step 8 of the pattern?
(e) What step of the pattern would need 30 matches?

3. Find the rules for the symbols shown in the following tables.

(a)

s	m
1	5
2	6
3	7
4	8
5	9
6	10

(b)

A	B
1	5
2	10
3	15
4	20
5	25
6	30

(c)

p	q
20	10
21	11
22	12
23	13
24	14
25	15

4. Use the rule shown to answer the following questions.

$y = 10 \times x$

(a) Find y if: (i) $x = 1$ (ii) $x = 4$ (iii) $x = 7$
(b) Find x if: (i) $y = 20$ (ii) $y = 50$ (iii) $y = 100$

5. Copy and complete the table for the rule below.

$B = A + 6$

A	B
1	
3	
	13
9	
	25

6. Sunil was making a fruit drink using 3 cups of orange juice to 1 cup of pineapple juice.

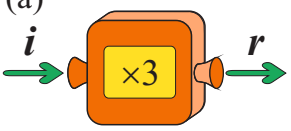
(a) Copy and complete this table showing the number of cups of orange juice, o, needed for p cups of pineapple juice.

p	o
1	
2	
3	
4	

(b) Write a rule that can be used to find the number of cups of orange juice, o, needed for p cups of pineapple juice.

7. Copy and complete the tables for the number machines below.

(a)

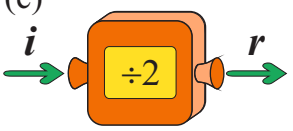


i	1	5		
r			21	33

(b)

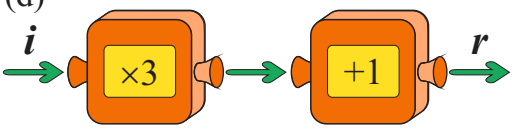
i	2	6		
r			15	21

(c)



i	2	8		
r			9	12

(d)



i	1	5	7		11	
r				31		37

8. Match the flowchart with the table of values.

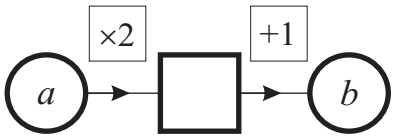
(a)

a	b
1	3
2	5
3	7
4	9

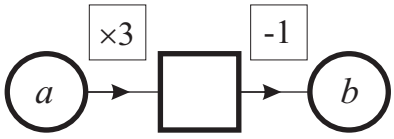
(b)

a	b
1	2
2	5
3	8
4	11

A



B



9. For the following tables:

(i) draw the flowchart.

(ii) find the equation.

(a)

x	y
1	5
2	7
3	9
4	11

(b)

N	M
1	1
2	4
3	7
4	10