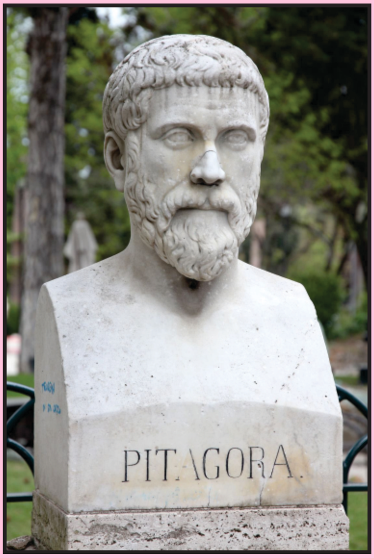
Pythagoras

Pythagoras was a Greek philosopher, mathematician and scientist.
(570-495 B.C.)
Pythagoras contributed to many fields including philosophy, astronomy, mathematics, music, politics and medicine.
His philosophies influenced many philosophers including Plato.
He founded the Pythagorean School of Mathematics and it was from this school that the well known Pythagoras' Theorem was developed.
Along with his contributions to the fields of science and mathematics his writings and poetry are well known.
Two of his quotes are shown below.



Rome, Italy. Bust statue of Pythagoras, famous philosopher, mathematician and scientist.
Sculpture in Villa Borghese park.

$$A^2 + B^2 = C^2$$

ABOVE ALL THINGS, RESPECT YOURSELF

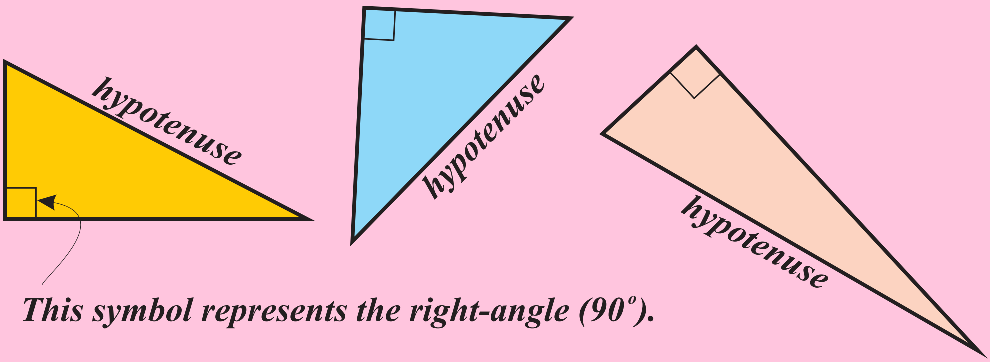
IN NO WAY NEGLECT THE HEALTH OF YOUR BODY

Pythagoras' Theorem

Remember:

- A right-angled triangle is a triangle that has one angle that is 90°.
- The longest side of a right-angled triangle is opposite the right-angle and is called the ***hypotenuse***.

Examples of right-angled triangles showing their hypotenuse.



This symbol represents the right-angle (90°).

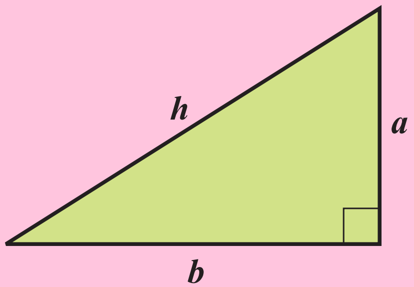
Pythagoras' Theorem states that:

For any ***right-angled triangle*** the square of the longest side (the ***hypotenuse***) is equal to the sum of the squares of the two shorter sides.

$$h^2 = a^2 + b^2$$

or

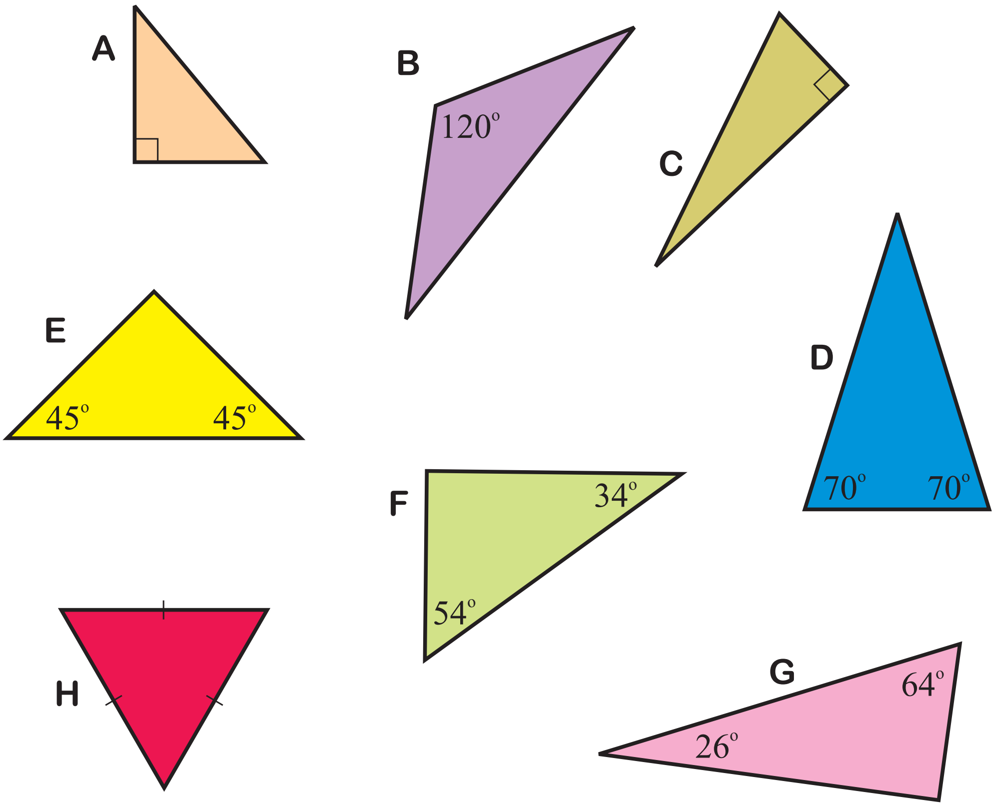
$$a^2 + b^2 = h^2$$



EXERCISE 13A

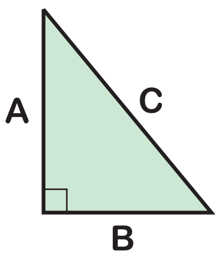
1. Which of the following triangles are ***right-angled triangles***?

Remember: The sum of the angles in a triangle is 180°

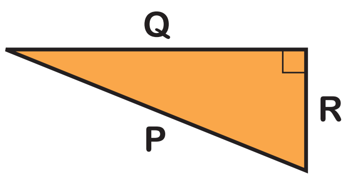


2. Which of the sides in each of the following triangles is the hypotenuse?

(a)



(b)



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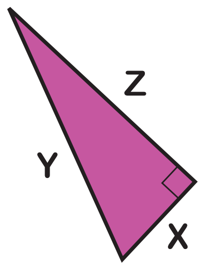
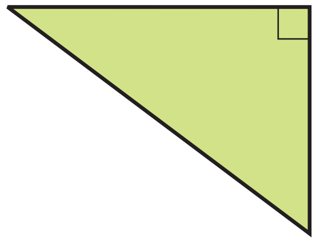
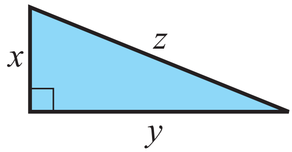


3. (a) Use a ruler to measure the length of the hypotenuse in this triangle.
(b) Use a ruler to measure the other two sides of this triangle.
(c) Show that this triangle obeys Pythagoras' Theorem.



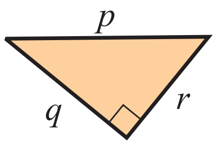
4. State whether the following statements are ***TRUE*** or ***FALSE***.
(a) In this triangle:

$$z^2 = x^2 + y^2$$



(b) In this triangle:

$$r^2 = p^2 + q^2$$



(c) The hypotenuse is always opposite the right-angle.
(d) The hypotenuse is always the longest side of a triangle.
(e) In this triangle:

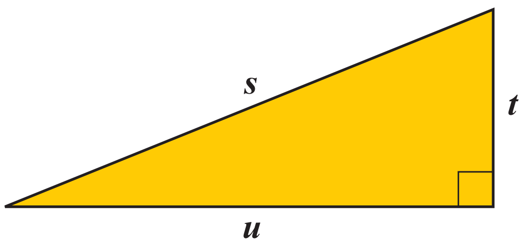
(i) $s^2 + t^2 + u^2 = 0$

(ii) $s^2 - t^2 - u^2 = 0$

(iii) $t^2 = s^2 - u^2$

(iv) $u^2 = s^2 - t^2$

(v) $s = \sqrt{t^2 + u^2}$



(f) It is impossible to have a right-angled equilateral triangle.
(g) It is impossible to have a right-angled isosceles triangle.
(h) It is impossible to have a right-angled scalene triangle.

A ***Pythagorean Triad*** is a set of three numbers that follow Pythagoras' theorem.

Examples

1. 3, 4, 5

If three numbers follow Pythagoras' Theorem then the sum of the squares of the two smaller numbers should equal the square of the larger number.

$3^2 + 4^2$ should equal 5^2

$3^2 + 4^2$	5^2
$= 9 + 16$	$= \mathbf{25}$
$= \mathbf{25}$	

These are equal so 3, 4, 5 is a Pythagorean Triad

2. 36, 39, 15

The two smaller numbers are 15 and 36.
For these three numbers to be a Pythagorean Triad then:

$15^2 + 36^2$ should equal 39^2

$15^2 + 36^2$	39^2
$= 225 + 1296$	$= \mathbf{1521}$
$= \mathbf{1521}$	

These are equal so 15, 36, 39 is a Pythagorean Triad

3. 9, 43, 40

The two smaller numbers are 9 and 40.
For these three numbers to be a Pythagorean Triad then:

$9^2 + 40^2$ should equal 43^2

$9^2 + 40^2$	43^2
$= 81 + 1600$	$= \mathbf{1849}$
$= \mathbf{1681}$	

These are not equal so 9, 43, 40 is not a Pythagorean Triad

5. Use a calculator to find the following.

(a) 18^2 (b) 6.5^2 (c) 13.4^2 (d) 8.72^2

6. Which of the following sets of numbers are Pythagorean Triads?

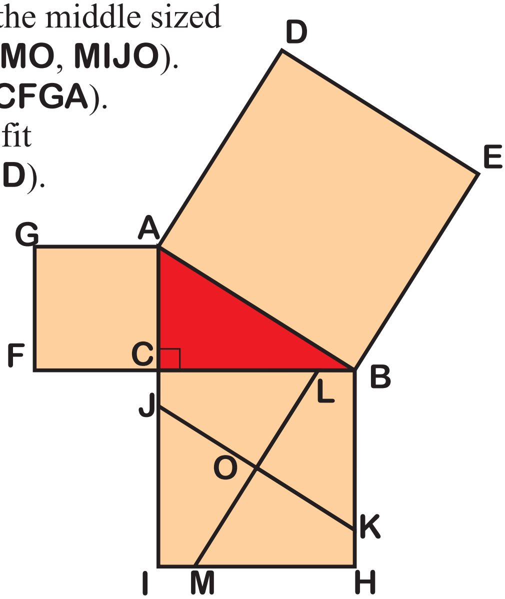
(a) 9, 40, 41 (b) 16, 63, 65 (c) 11, 60, 63
(d) 15, 112, 113 (e) 13, 83, 85 (f) 264, 23, 265
(g) 25, 7, 24 (h) 82, 18, 79 (i) 164, 160, 36
(j) 131, 78, 105 (k) 50, 48, 14 (l) 224, 305, 207

7. Find the three numbers from the following list that form a Pythagorean Triad.

78 301 317 321 308 75

8. There are many ways to prove Pythagoras' Theorem.
Here is one of them.
Make this construction on paper that you can cut.

Step 1 - Draw a right-angled triangle **ABC**.
Step 2 - Construct three ***squares*** on the sides of the triangle (**ABED**, **CBHI** and **ACFG**).
Step 3 - Find the ***centre*** of the middle sized square (**O**).
Step 4 - Draw a line (**JK**) through **O** that is parallel to **AB**.
Step 5 - Draw a line (**LM**) through **O** that is parallel to **EB**.
Step 6 - Cut out the four sections of the middle sized square (**JCLO**, **LBKO**, **KHMO**, **MIJO**).
Step 7 - Cut out the smaller square (**CFGA**).
Step 8 - Arrange these five pieces to fit into the largest square (**ABED**).



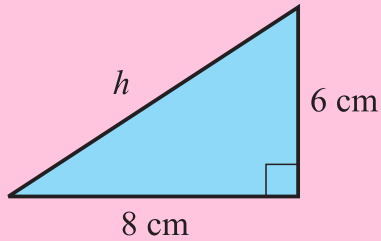
Explain how this proves Pythagoras' Theorem.

Finding the Hypotenuse

EXERCISE 13B

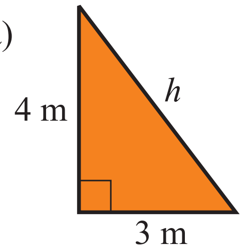
Example 1 Find the length of the hypotenuse in the following right-angled triangle.

$h^2 = a^2 + b^2$
$h^2 = 6^2 + 8^2$
$h^2 = 36 + 64$
$h^2 = 100$
$h = \sqrt{100}$
$\boldsymbol{h = 10}$ **cm**

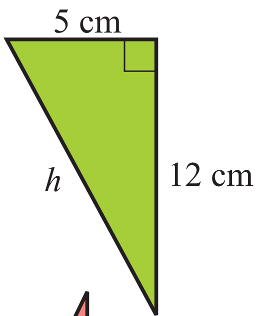


1. Find the length of the hypotenuse in each of the following triangles.

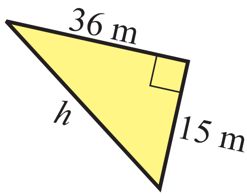
(a)



(b)



(c)



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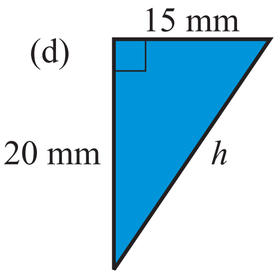

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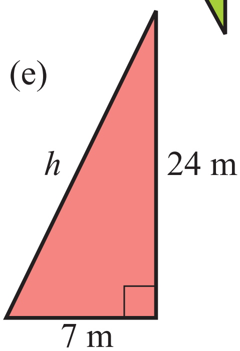

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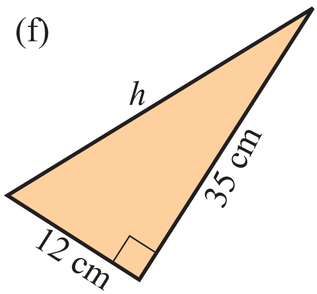

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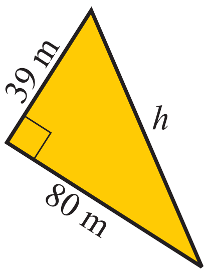
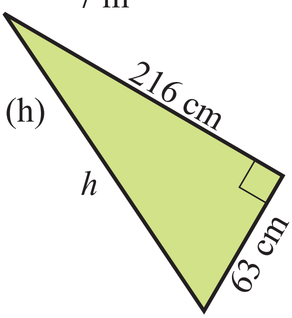
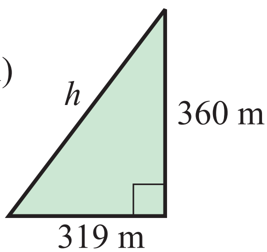


(h)

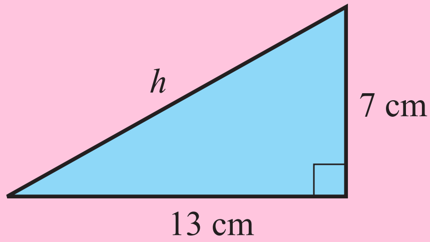


(i)



Example 2 Find the length of the hypotenuse in the following right-angled triangle.

$h^2 = a^2 + b^2$
$h^2 = 13^2 + 7^2$
$h^2 = 169 + 49$
$h^2 = 218$
$h = \sqrt{218}$



Note this answer is a surd (irrational number) and an exact answer cannot be calculated. The answer can be given two ways:
(a) as a surd
(b) as a decimal number correct to a given number of decimal places.

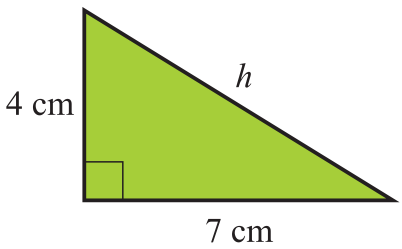
In this example the answer would be:

(a) $\boldsymbol{h = \sqrt{218}}$ **cm**

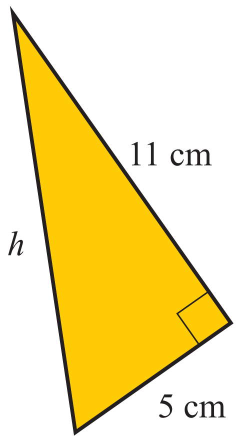
(b) $h = \sqrt{218}$
$= 14.764823......$
$\boldsymbol{h}$ = **14.8 cm** (one decimal place)

2. Find the length of the hypotenuse in the following right-angled triangles.
Give answers as: (i) surds
(ii) decimal numbers correct to one decimal place

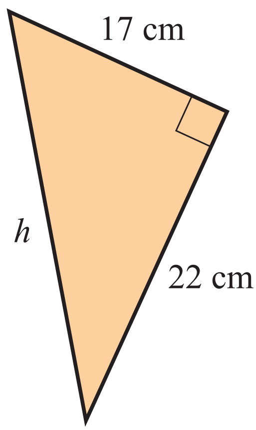
(a)



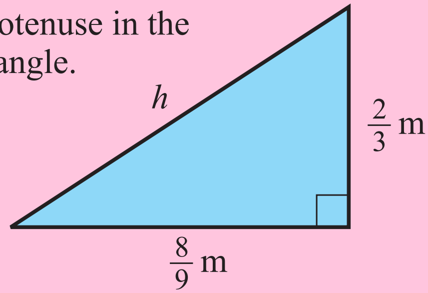
(b)



(c)



Example 3 Find the length of the hypotenuse in the following right-angled triangle.



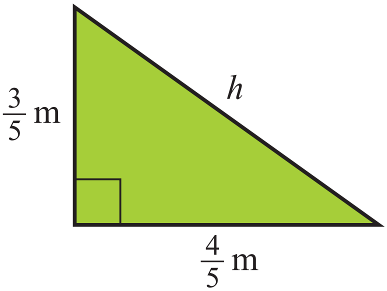
$$h^2 = a^2 + b^2$$
$$h^2 = \left(\frac{8}{9}\right)^2 + \left(\frac{2}{3}\right)^2$$
$$h^2 = \frac{64}{81} + \frac{4}{9}$$
$$h^2 = \frac{64}{81} + \frac{36}{81}$$

Common denominator so the fractions can be added

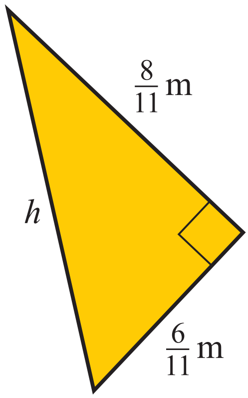
$$h^2 = \frac{100}{81}$$
$$h = \sqrt{\frac{100}{81}}$$
$$\boldsymbol{h = \frac{10}{9}\ \text{m}}$$

3. Find the length of the hypotenuse in the following right-angled triangles.

(a)



(b)



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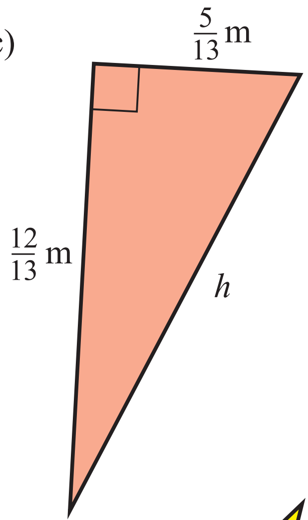

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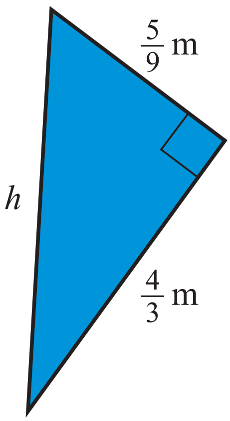

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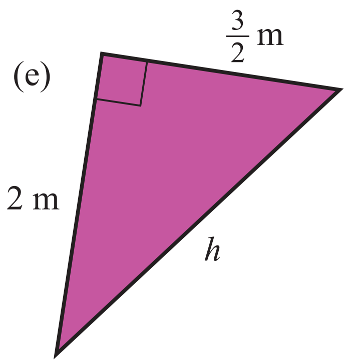

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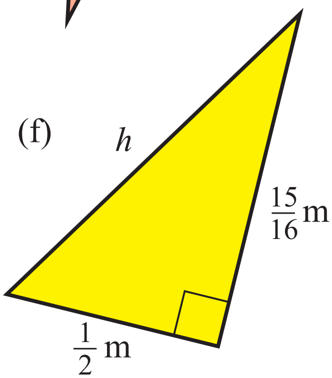
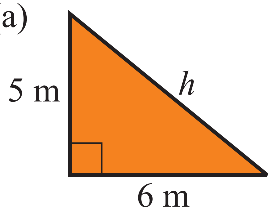
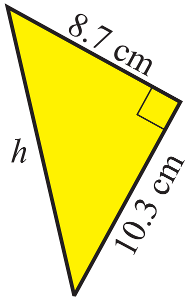
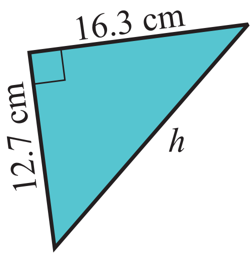


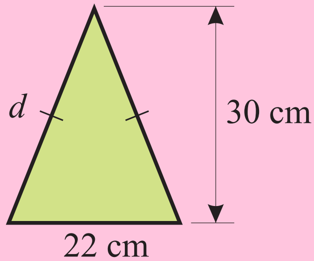
4. Find the length of the hypotenuse in each of the following triangles. Give answers correct to one decimal place.

(a) 

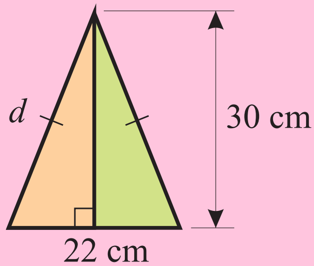
(b) 

(c) 

Example Find the unknown length in the following triangle. Give answer correct to one decimal place.



To find the unknown length a right-angled triangle needs to be formed to use Pythagoras' Theorem.



$d^2 = a^2 + b^2$

$d^2 = 11^2 + 30^2$

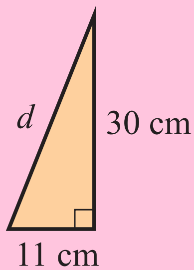
$d^2 = 121 + 900$

$d^2 = 1021$

$d = \sqrt{1021}$

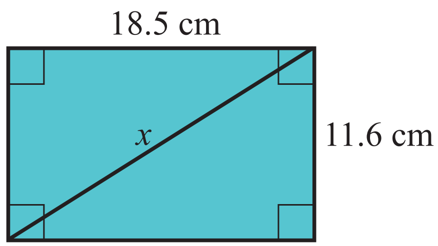
$d = 31.95309...$

$\mathbf{d = 32.0}$ **cm**



5. Find the unknown lengths in the following shapes.
Give answers correct to one decimal place.

(a)



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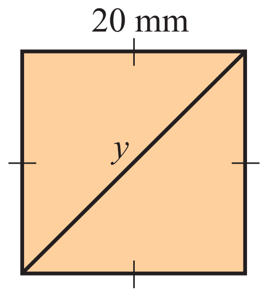

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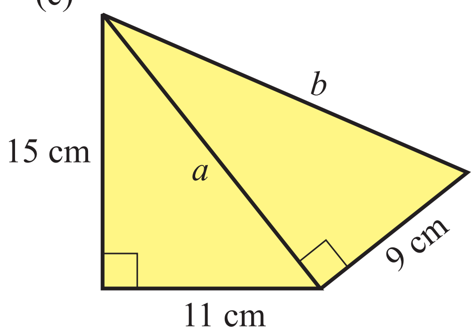

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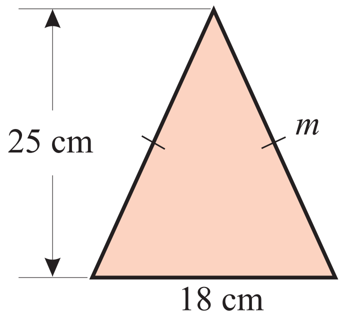

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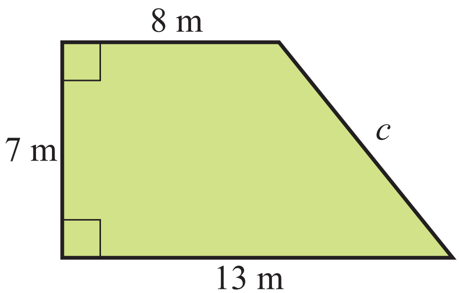

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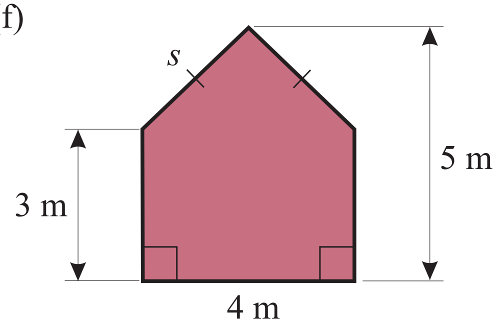


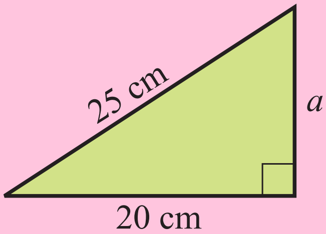
Finding One of the Shorter Sides

EXERCISE 13C

Example Find the length of the unknown side in the following right-angled triangle.

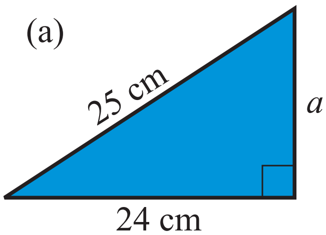
Note - use $\boldsymbol{a^2 + b^2 = h^2}$ when finding a shorter side

$a^2 + b^2 = h^2$
$a^2 + 20^2 = 25^2$
$a^2 + 400 = 625$
$a^2 = 625 - 400$
$a^2 = 225$
$a = \sqrt{225}$
$\boldsymbol{a = 15}$ **cm**



1. Find the length of the unknown side in each of the following triangles.

(a)



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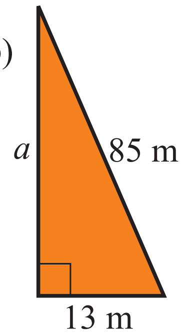

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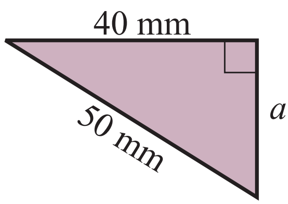

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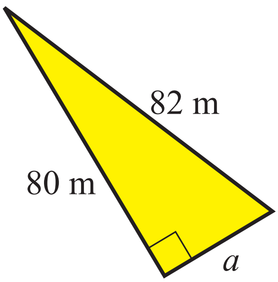

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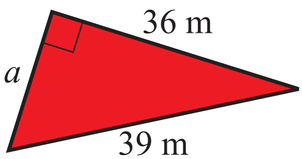

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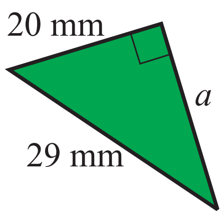
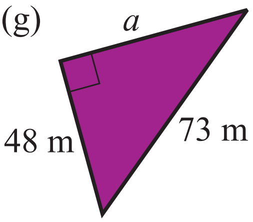
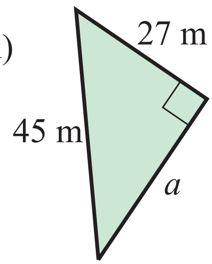


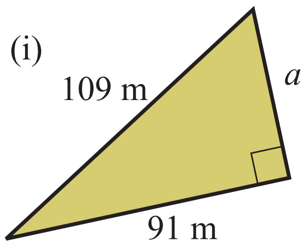
(g)



(h)

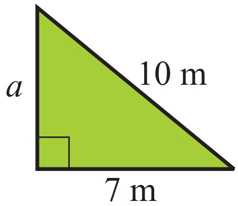


(i)

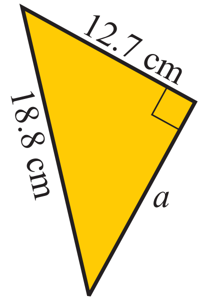


2. Find the length of the unknown side in each of the following triangles. Give answers correct to one decimal place.

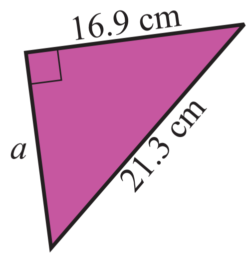
(a)



(b)

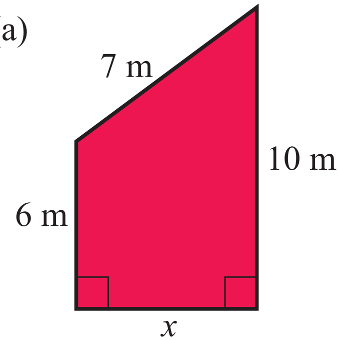


(c)

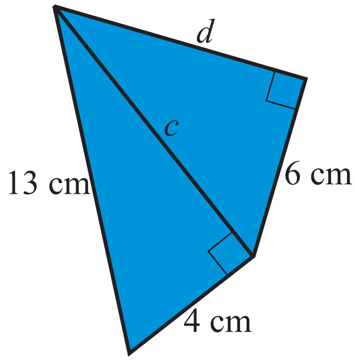


3. Find the unknown lengths in the following shapes. Give answers correct to one decimal place.

(a)



(b)



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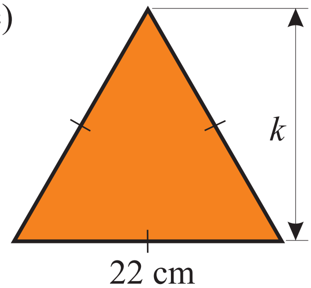
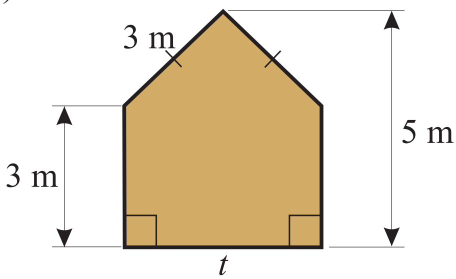
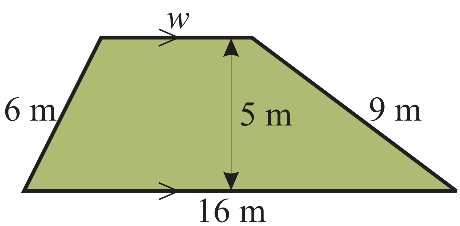


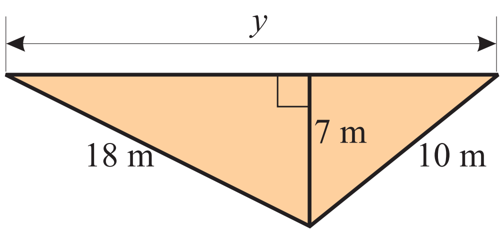
(d)



(e)



(f)



Mixed Problems

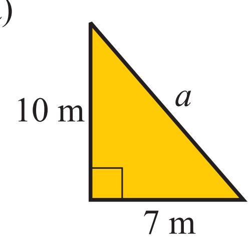
EXERCISE 13D

1. Calculate the length of a diagonal of a square with side length 25 cm. Give answer correct to one decimal place.

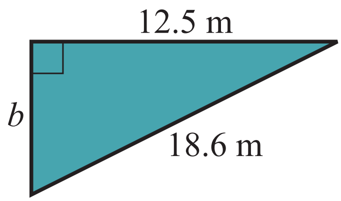
2. Calculate the width of a rectangle that is 208 cm long and has a diagonal that is 233 cm long.

3. Find the unknown lengths in the following shapes. Give answers correct to one decimal place.

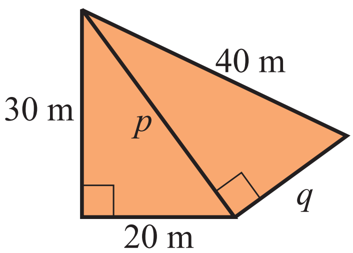
(a)



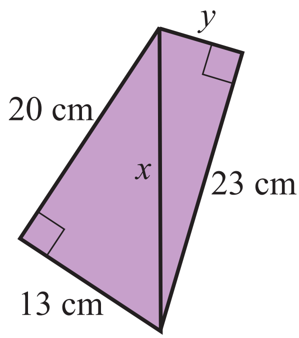
(b)



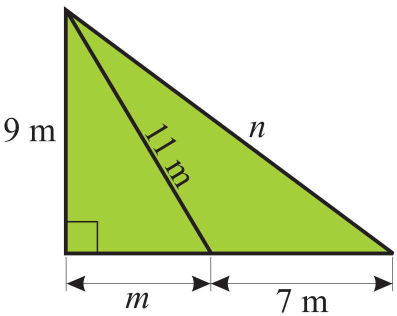
(c)



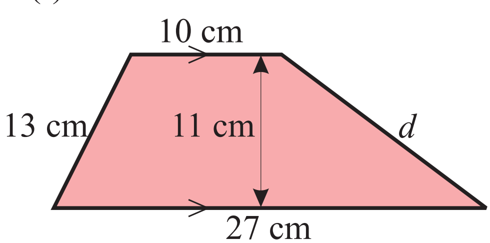
(d)



(e)



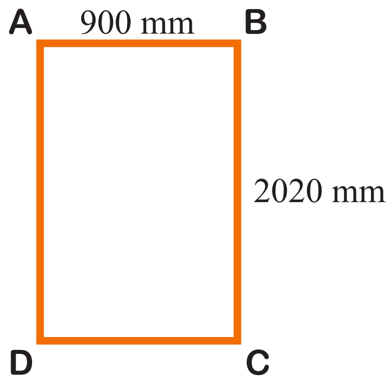
(f)



Practical Problems

EXERCISE 13E

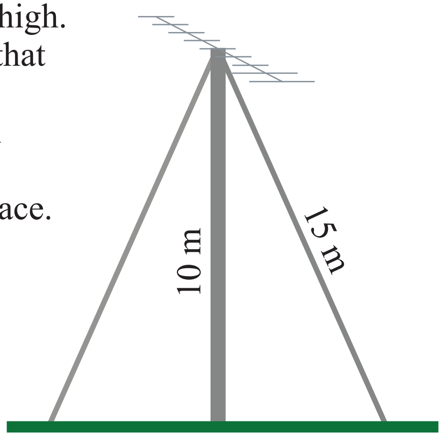
1. A carpenter wants to check if a door frame (**ABCD**) is 'square' (all angles are 90°) by measuring the diagonals. These will be equal only if all the angles of the frame are 90°.
The dimensions of the door frame are 2020 mm × 900 mm as shown.



(a) What should be the length of each diagonal (**AC** & **BD**)?
Give answer to the nearest mm.

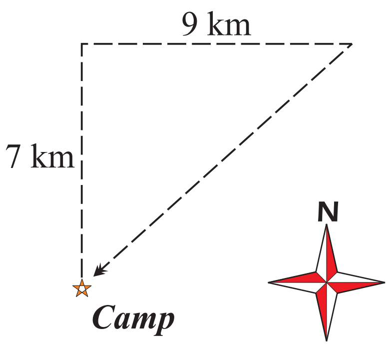
(b) The carpenter found that **AC** measured 2230 mm.
Which of the angles (**A**, **B**, **C**, **D**) are ***greater*** than 90°?

2. (a) An antenna support pole is 10 metres high. The pole is to be supported by cables that are 15 metres long.
How far from the base of the pole will the cables be fixed to the ground?
Give answer correct to one decimal place.

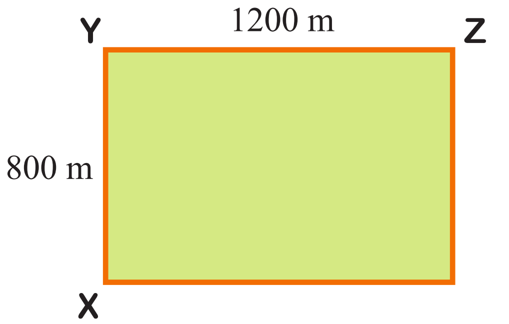


(b) It was found that the pole was still unstable. More cables were fixed from the top of the pole to points 25 metres from the base of the pole.
What will be the length of these cables?
Give answer correct to one decimal place.

3. Two bush walkers hiked 7 km north from their camp, then 9 km east and then walked in a straight line back to their camp.
How far did they hike?
Give answer correct to one decimal place.

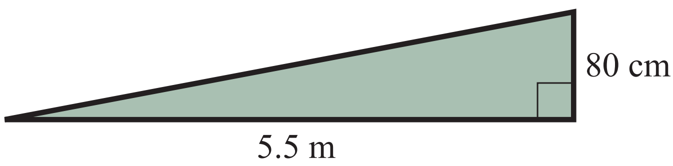


4. Noel and Paddy were at one corner (**X**) of a rectangular paddock. They wanted to run to the opposite corner (**Z**).
Noel ran around the paddock (**X-Y-Z**) at a speed of 8 metres per second.
Paddy ran straight from **X** to **Z**, through the paddock, at a speed of 5 metres per second.



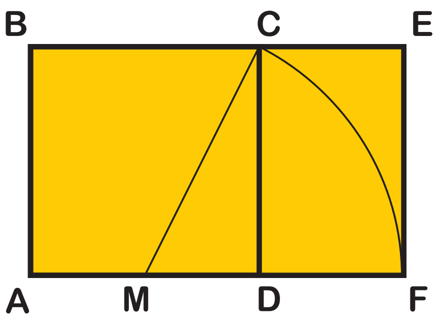
(a) How far did Paddy have to run (to the nearest metre)?
(b) How many seconds did it take each runner to get to point **Z**?

5. A ramp for wheelchair access to a building is to be built.
The ramp is to start 5.5 metres from the building and finish at the doorway which is 80 cm above the ground.
How long is the ramp?
Give answer in metres correct to three decimal places.



6. An artist wants to make a rectangular frame for a painting using the ***Golden Ratio***. A rectangle with this shape is said to be the most pleasing to look at and is called a ***Golden Rectangle***.
To achieve this she must follow these steps:

Step 1 - Draw a square **ABCD**.
Step 2 - Mark a point (**M**) midway along **AD**.
Step 3 - Connect **M** to **C** and draw an arc using **M** as the centre to find point **F**.



Step 4 - **ABEF** is called a ***Golden Rectangle***.

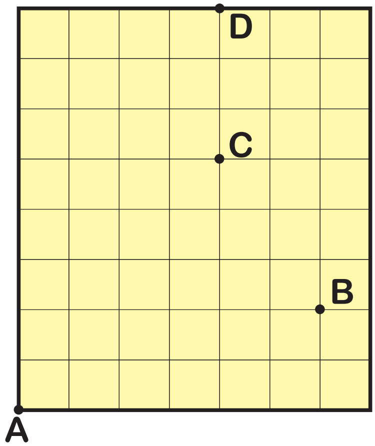
Find the length (**AF**) of the artist's frame if it is to be 70 cm wide (**AB**).
Give answer correct to one decimal place.

7. A Royal Flying Doctor has to visit three cattle stations. The map shown here indicates the position of the airport (**A**) and the three stations (**B**, **C** and **D**). ***The lines on the grid are 50 km apart.*** The doctor needs to visit station **B** first but needs to work out which will be the shortest route to visit all the stations and return to the airport:

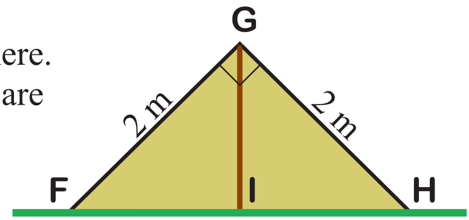
(a) **A-B-C-D-A** or

(b) **A-B-D-C-A**.

Calculate the total length of each route to find out which is the shortest. Give answers correct to one decimal place.



8. The end view of a tent is shown here. The sides of the tent (**FG** & **GH**) are 2 metres and are at right angles. Find the height of the tent post (**GI**). Give answer correct to three decimal places.



9. (a) Complete this saying:

The shortest distance between two points is:

A ___A___T _I__

(b) Rearrange the letters of each of the words below to find a saying that means to travel in a straight line between two places.

SA HET WORC SLIFE

Pythagoras in 3D

EXERCISE 13F

Example

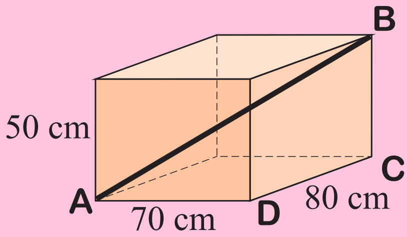
Find the length of the diagonal **AB** in the box shown.

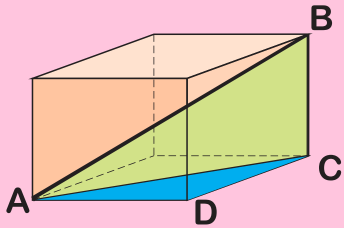
Step 1 - Identify **AB** as the hypotenuse of triangle **ABC**.

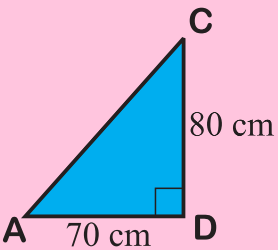
Step 2 - Identify **AC** as the hypotenuse of triangle **ACD**.

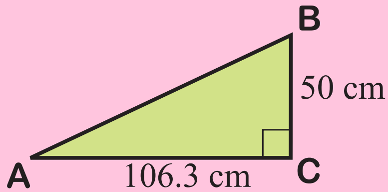
Step 3 - Calculate **AC**.

Step 4 - Calculate **AB**.









$AC^2 = 70^2 + 80^2$

$AC^2 = 11300$

$AC = 106.3$ cm

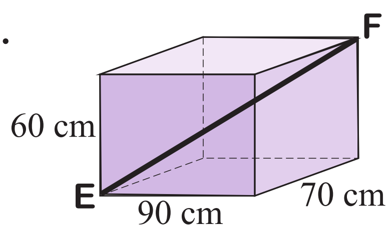
$AB^2 = 106.3^2 + 50^2$

$AB^2 = 13800$

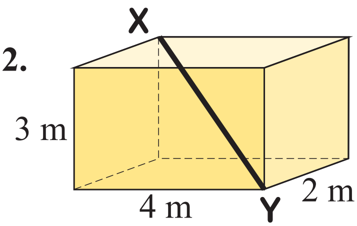
$\mathbf{AB = 117.5}$ **cm**

Find the length of the diagonals shown in the cuboids below.
Give answers correct to one decimal place.

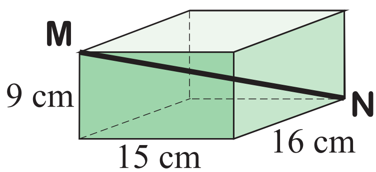
1.



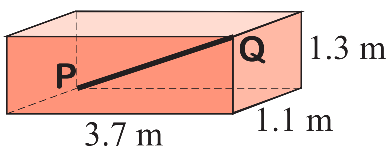
2.



3.

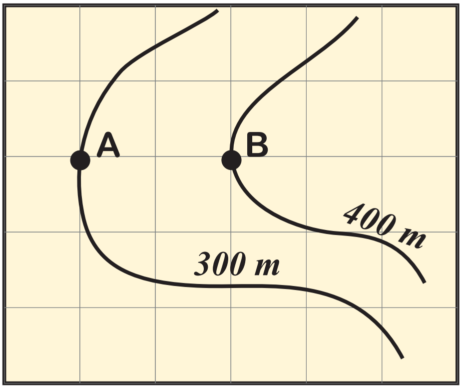


4.



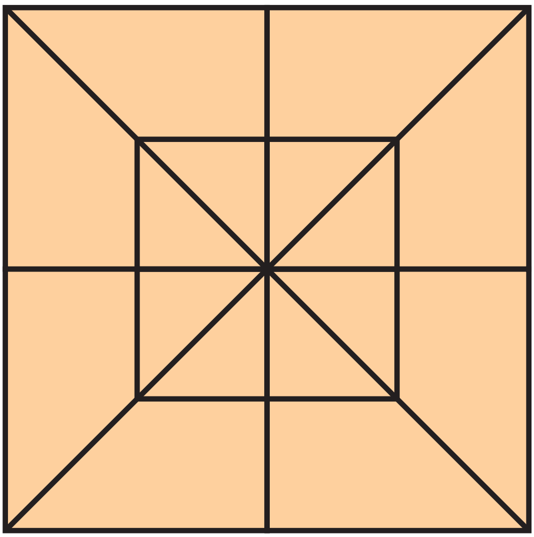
PROBLEM SOLVING

A contour map is shown here.
The grid on the map is 100 m squares.
Heidi is walking from **A** to **B**.
Point **A** is 300 m above sea level.
Point **B** is 400 m above sea level.
There is a constant slope between points **A** and **B**.
Use the grid and Pythagoras' Theorem to calculate the distance Heidi has to walk between points **A** and **B**.
Give correct to one decimal place.



PUZZLE

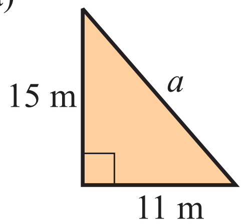
How many right-angled triangles are in this shape?



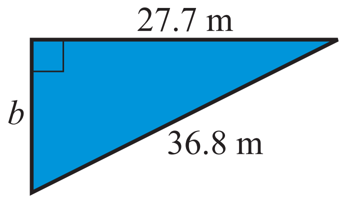
CHAPTER REVIEW

1. Find the unknown lengths in the following shapes.
Give answers correct to one decimal place.

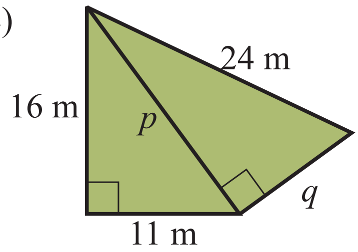
(a)



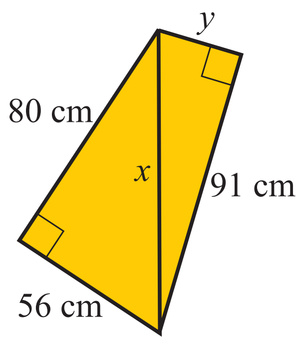
(b)



(c)

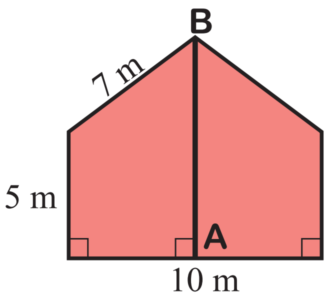


(d)



2. A yacht sails 15 km north from the yacht club then 12.5 km west.
How far is the yacht from the yacht club?
Give answer correct to one decimal place.

3. Find the length of **AB**.
Give answer correct to one decimal place.



4. Find the length of the **XY** in the cuboid below.
Give answer correct to one decimal place.