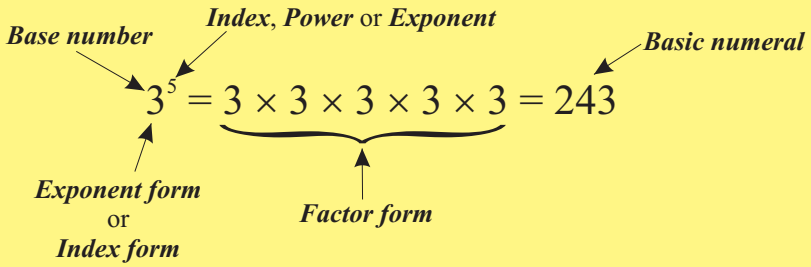
Index Laws

Index Laws

Example



Note: the ***plural*** of ***index*** is ***indices***

EXERCISE 6A

1. Write the following in factor form.

Example $4^8 = \mathbf{4} \times \mathbf{4} \times \mathbf{4} \times \mathbf{4} \times \mathbf{4} \times \mathbf{4} \times \mathbf{4} \times \mathbf{4}$

(a) 2^4 (b) 6^7 (c) 10^2 (d) 9^8 (e) 8^{11}

2. Write the following in index form.

Example $5 \times 5 \times 5 \times 5 \times 5 \times 5 = \mathbf{5^6}$

(a) $6 \times 6 \times 6 \times 6$
(b) $9 \times 9 \times 9 \times 9 \times 9 \times 9$
(c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
(d) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

3. Write the following in factor form then combine into index form.

Example

$5^3 \times 5^4$

$= \underbrace{5 \times 5 \times 5}_{} \times \underbrace{5 \times 5 \times 5 \times 5}_{}$

$= \mathbf{5^7}$

(a) $3^2 \times 3^4$ (b) $2^3 \times 2^2$ (c) $7^5 \times 7^3$ (d) $6^3 \times 6^5$
(e) $5^3 \times 5^3$ (f) $10^2 \times 10^6$ (g) $9^4 \times 9^3$ (h) $8^6 \times 8^5$

4. Can you think of a way of finding the answers in question 3 without writing the factors?

Index Law 1

Index Law 1 states:

If terms with the ***same base number*** are multiplied, the indices can be ***added***.

Note: It is not necessary to show an index of 1.
$8 = 8^1$, $4 = 4^1$, $2 = 2^1$, $12 = 12^1$, etc

Examples

1. $4^6 \times 4^7$
$= 4^{(6+7)}$
$= \mathbf{4^{13}}$

2. $7^8 \times 7^6$
$= 7^{(8+6)}$
$= \mathbf{7^{14}}$

3. $6^4 \times 6$
$= 6^4 \times 6^1$
$= 6^{(4+1)}$
$= \mathbf{6^5}$

4. 9×9^{14}
$= 9^1 \times 9^{14}$
$= 9^{(1+14)}$
$= \mathbf{9^{15}}$

5. Use Index Law 1 to find the answers to the following in index form.

(a) $6^3 \times 6^7$ (b) $5^4 \times 5^8$ (c) $4^9 \times 4^2$ (d) $8^6 \times 8$
(e) $2^{10} \times 2^5$ (f) $13^6 \times 13^8$ (g) 3×3^8 (h) $15^7 \times 15^4$
(i) $9^{14} \times 9^{13}$ (j) $17^8 \times 17^{14}$ (k) 11×11^{12} (l) $26^{15} \times 26$

EXERCISE 6B

1. Write the following in factor form then simplify and write the answer in index form.

Example

$$\frac{6^5}{6^2}$$

$$= \frac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6}$$

$$= \frac{6 \times 6 \times 6 \times \cancel{6} \times \cancel{6}}{\cancel{6} \times \cancel{6}}$$

The two 6's on the denominator (bottom line) can be cancelled with two 6's on the numerator (top line).

$$= \frac{6 \times 6 \times 6}{1}$$

$$= 6 \times 6 \times 6$$

$$= \mathbf{6^3}$$

(a) $\frac{4^5}{4^2}$ (b) $\frac{2^6}{2^4}$ (c) $\frac{7^7}{7^3}$ (d) $\frac{9^6}{9^2}$

(e) $\frac{5^8}{5^5}$ (f) $\frac{10^7}{10}$ (g) $\frac{3^6}{3^3}$ (h) $\frac{6^9}{6^2}$

(i) $\frac{8^8}{8^6}$ (j) $\frac{11^9}{11^5}$ (k) $\frac{2^5}{2}$ (l) $\frac{13^4}{13^3}$

(m) $\frac{3^5}{3^4}$ (n) $\frac{15^7}{15^5}$ (o) $\frac{20^4}{20}$ (p) $\frac{7^9}{7^7}$

2. Can you think of a way of finding the answers in question 1 without writing the factors?

Index Law 2

Index Law 2 states:

If terms with the ***same base number*** are divided, the index on the denominator (bottom line) is ***subtracted from*** the index on the numerator (top line).

Examples

1. $\dfrac{7^8}{7^6}$

$= 7^{(8-6)}$

$= \mathbf{7^2}$

2. $\dfrac{9^{12}}{9^5}$

$= 9^{(12-5)}$

$= \mathbf{9^7}$

3. $\dfrac{8^7}{8^6}$

$= 8^{(7-6)}$

$= 8^1$

$= \mathbf{8}$

4. $\dfrac{3^7}{3}$

$= \dfrac{3^7}{3^1}$

$= 3^{(7-1)}$

$= \mathbf{3^6}$

3. Use Index Law 2 to find the answers to the following in index form.

(a) $\dfrac{5^7}{5^2}$ (b) $\dfrac{3^9}{3^5}$ (c) $\dfrac{4^8}{4^3}$ (d) $\dfrac{7^6}{7^4}$

(e) $\dfrac{6^{13}}{6^5}$ (f) $\dfrac{12^{15}}{12^9}$ (g) $\dfrac{2^8}{2}$ (h) $\dfrac{8^9}{8^8}$

(i) $\dfrac{9^{12}}{9^{11}}$ (j) $\dfrac{15^9}{15}$ (k) $\dfrac{9^5}{9}$ (l) $\dfrac{17^{19}}{17^{13}}$

4. Use Index Law 2 to find the answers to the following in index form.

(a) $9^8 \div 9^5$ (b) $12^9 \div 12^3$

(c) $7^{10} \div 7^7$ (d) $8^{13} \div 8^{11}$

(e) $10^8 \div 10^7$ (f) $6^8 \div 6$

(g) $5^{15} \div 5^9$ (h) $2^{21} \div 2^{13}$

(i) $3^9 \div 3$ (j) $20^{17} \div 20^{16}$

(k) $4^{13} \div 4^{12}$ (l) $17^{16} \div 17^5$

Example $6^7 \div 6^4$

$= \dfrac{6^7}{6^4}$

$= 6^{(7-4)}$

$= \mathbf{6^3}$

EXERCISE 6C

1. Write the following in factor form then simplify by cancelling.

(a) $\frac{5^4}{5^4}$ (b) $\frac{3^7}{3^7}$ (c) $\frac{4^3}{4^3}$ (d) $\frac{7^5}{7^5}$

2. Use Index Law 2 to find the answers to the problems in question 1.

3. Use the answers from questions 1 and 2 to complete the following:

(a) $5^0 =$ ___ (b) $3^0 =$ ___ (c) $4^0 =$ ___ (d) $7^0 =$ ___

Index Law 3

Index Law 3 states:

If a term has an index of **0** then the answer is **1**.

$7^0 = 1$, $3^0 = 1$, $27^0 = 1$, $123^0 = 1$, etc

4. Evaluate the following.

Examples

1. $6^0 + 8$
$= 1 + 8$
$= \mathbf{9}$

2. $7^0 + 2^2 - 8^0$
$= 1 + 4 - 1$
$= \mathbf{4}$

3. $(7 + 5)^0$
$= 12^0$
$= \mathbf{1}$

4. $\frac{5^0 + 17}{2^3 + 9^0}$
$= \frac{1 + 17}{8 + 1}$
$= \frac{18}{9}$
$= \mathbf{2}$

(a) $7 - 8^0$ (b) $12 + 7^0$ (c) $3^0 + 3^1 + 3^2$ (d) $(7 + 4)^0$

(e) $(4 + 8^0)^2$ (f) 5×4^0 (g) $(124 - 87)^0$ (h) $(7^5 + 9^4)^0$

(i) $\frac{9 + 18}{3^0}$ (j) $\frac{4^2 - 7^0}{2^2 - 2^0}$ (k) $\frac{(4 + 2)^2}{8^0 + 11}$ (l) $\frac{(6^2 + 3^2) \times 7^0}{(5 - 4^0)^2 - 6^0}$

EXERCISE 6D

1. Write the following in factor form then combine into index form.

Example $(5^3)^4$

$= 5^3 \times 5^3 \times 5^3 \times 5^3$

$= \underbrace{5 \times 5 \times 5} \times \underbrace{5 \times 5 \times 5} \times \underbrace{5 \times 5 \times 5} \times \underbrace{5 \times 5 \times 5}$

$= \mathbf{5^{12}}$

(a) $(4^2)^4$ (b) $(8^3)^2$ (c) $(6^5)^2$ (d) $(3^2)^5$

(e) $(2^4)^4$ (f) $(9^2)^2$ (g) $(5^4)^3$ (h) $(10^3)^5$

(i) $(12^2)^6$ (j) $(7^7)^2$ (k) $(3^2)^7$ (l) $(2^3)^6$

2. Can you think of a way to find the answers in question 1 without writing the factors?

Index Law 4

Index Law 4 states:

If a term with an index is raised to an index, the indices are ***multiplied***.

Examples

1. $(9^5)^6$
$= 9^{(5 \times 6)}$
$= \mathbf{9^{30}}$

2. $(3^7)^5$
$= 3^{(7 \times 5)}$
$= \mathbf{3^{35}}$

3. Use Index Law 4 to find the answers to the following in index form.

(a) $(5^6)^4$ (b) $(6^5)^5$ (c) $(2^7)^4$ (d) $(9^7)^8$

(e) $(7^8)^5$ (f) $(8^4)^9$ (g) $(10^5)^{10}$ (h) $(11^9)^6$

(i) $(13^7)^6$ (j) $(9^{12})^6$ (k) $(3^9)^7$ (l) $(8^{20})^8$

Index Law 5

Index Law 5 states:

If any number of terms that are multiplied or divided are all raised to the same index, each term is raised to that index.

Examples

1. $(3^6 \times 5^2)^4$

$= (3^6)^4 \times (5^2)^4$

$= \mathbf{3^{24} \times 5^8}$

2. $= \left(\frac{4^2}{8^9}\right)^3$

$= \frac{(4^2)^3}{(8^9)^3}$

$= \mathbf{\frac{4^6}{8^{27}}}$

3. $(7^3 \div 9^5)^6$

$= \left(\frac{7^3}{9^5}\right)^6$

$= \frac{(7^3)^6}{(9^5)^6}$

$= \mathbf{\frac{7^{18}}{9^{30}}}$

EXERCISE 6E

1. Expand the following terms.

(a) $(3^5 \times 4^2)^6$ (b) $(7^4 \times 2^8)^3$

(c) $(6^7 \times 9^4)^7$ (d) $(5^8 \times 8^9)^6$

Example $(5^3 \times 7^6)^2$

$= \mathbf{5^6 \times 7^{12}}$

2. Expand the following terms.

(a) $\left(\frac{3^6}{8^3}\right)^7$ (b) $\left(\frac{5^9}{6^5}\right)^4$

(c) $(4^3 \div 9^{11})^5$ (d) $(2^6 \div 7^8)^{12}$

Examples

1. $\left(\frac{7^5}{2^4}\right)^6$

$= \mathbf{\frac{7^{30}}{2^{24}}}$

2. $(9^7 \div 5^8)^2$

$= \left(\frac{9^7}{5^8}\right)^2$

$= \mathbf{\frac{9^{14}}{5^{16}}}$

Example

Simplify $\frac{6^3}{6^5}$

This can be done two ways.

1. Use Index Law 2.

$$\frac{6^3}{6^5}$$
$$= 6^{(3-5)}$$
$$= \mathbf{6^{-2}}$$

2. Write in factor form and cancel.

$$\frac{6^3}{6^5}$$
$$= \frac{6 \times 6 \times 6}{6 \times 6 \times 6 \times 6 \times 6}$$
$$= \frac{\cancel{6} \times \cancel{6} \times \cancel{6}}{6 \times 6 \times \cancel{6} \times \cancel{6} \times \cancel{6}}$$
$$= \frac{1}{6 \times 6}$$
$$= \frac{\mathbf{1}}{\mathbf{6^2}}$$

It can be seen from these two answers that: $\mathbf{6^{-2} = \frac{1}{6^2}}$

This is Index Law 6

Index Law 6

Index Law 6 states:

A term with a negative index can be written on the denominator with a positive index.

Example $6^{-2} = \frac{1}{6^2}$

EXERCISE 6F

1. Write the following with positive indices.

(a) 6^{-6} (b) 8^{-9} (c) 4^{-13} (d) 5^{-7} (e) 12^{-8}

2. Write the following terms with positive indices.

Examples

1. 3×7^{-9}

$= 3 \times \frac{1}{7^9}$

$= \frac{3}{7^9}$

2. $6^4 \times 5^{-3}$

$= 6^4 \times \frac{1}{5^3}$

$= \frac{6^4}{5^3}$

(a) 3×7^{-9} (b) 8×2^{-5} (c) $9^{-4} \times 3$ (d) $4^{-6} \times 2$

(e) $6^2 \times 4^{-8}$ (f) $12^3 \times 3^{-8}$ (g) $7^{-2} \times 4^6$ (h) $8^{-10} \times 7^{12}$

3. Write the following terms with positive indices.

A term in the denominator with a negative index is the same as that term in the numerator with a positive index.

Examples

1. $\frac{1}{4^{-7}}$

$= \mathbf{4^7}$

2. $\frac{7}{2^{-8}}$

$= \mathbf{7 \times 2^8}$

3. $\frac{9}{4 \times 7^{-2}}$

$= \frac{\mathbf{9 \times 7^2}}{\mathbf{4}}$

4. $\frac{6 \times 4^{-3}}{2^{-9} \times 5}$

$= \frac{\mathbf{6 \times 2^9}}{\mathbf{5 \times 4^3}}$

(a) $\frac{1}{8^{-3}}$ (b) $\frac{1}{5^{-7}}$ (c) $\frac{4}{2^{-9}}$ (d) $\frac{3}{6^{-2}}$

(e) $\frac{4}{9 \times 8^{-4}}$ (f) $\frac{7}{3 \times 2^{-9}}$ (g) $\frac{5}{7^{-3} \times 6}$ (h) $\frac{8}{4^{-5} \times 11}$

(i) $\frac{7^{-2}}{6 \times 5^{-8}}$ (j) $\frac{5 \times 7^{-2}}{4 \times 6^{-3}}$ (k) $\frac{3^5 \times 5^{-4}}{9^8 \times 7^{-2}}$ (l) $\frac{4^{-7} \times 2^6}{3^2 \times 9^{-8}}$

EXERCISE 6G

Use the Index Laws to simplify the following.
Write the answers in index form with positive indices.

Examples

1. $\dfrac{(2^7 \times 2^5)^4}{(2^6 \times 2^9)^3}$

$= \dfrac{(2^{12})^4}{(2^{15})^3}$

$= \dfrac{2^{48}}{2^{45}}$

$= \mathbf{2^3}$

2. $\dfrac{(6^7 \times 6^{-9})^{-4}}{(6^{-3})^5}$

$= \dfrac{(6^{-2})^{-4}}{6^{-15}}$

$= \dfrac{6^8}{6^{-15}}$

$= \mathbf{6^{23}}$

1. $\dfrac{(4^5 \times 4^6)^3}{(4^7 \times 4^3)^2}$

2. $\dfrac{(7^{-3} \times 7^6)^5}{(7^7 \times 7^0)^2}$

3. $\dfrac{8^{13}}{8^7} \times \dfrac{8^{19}}{8^{17}}$

4. $\dfrac{(3^7 \times 3^9)^{-3}}{(3^{14} \times 3^{10})^{-2}}$

5. $\dfrac{(5^{-8} \times 5^{-6})^{-4}}{(5^{11} \times 5^{16})^0}$

6. $\dfrac{8^{13} \times 8^{-6}}{(8^{-7})^2} \times \dfrac{8^{13}}{8^{29}}$

7. $\dfrac{(9^{-7})^8}{(9^{12} \times 9^3)^3} \times \dfrac{(9^{20})^5}{(9^{-7} \times 9^5)^9}$

8. $\dfrac{(6^7 \times 6^{13})^{-3}}{(6^4 \times 6^{10})^{-4}} \times \dfrac{(6^{16} \times 6)^5}{(6 \times 6^{10})^7}$

9. $\dfrac{(9^{12} \times 9^{-3})^3 \times 9^{-5}}{(9^3)^{-4}}$

10. $\dfrac{(8^{-7} \times 8^{-3})^{-2}}{(8^4 \times 8^{-7})^{-3}}$

11. $(5^{-3} \times 5^{-4})^2 \times (5^{-4})^3$

12. $\left(\dfrac{7^5}{7^{-2}}\right)^6 \times \left(\dfrac{7^{-8}}{7^3}\right)^{-7}$

13. $\left(\dfrac{8^5}{8^5}\right)^3 \times \left(\dfrac{8^{-7}}{8^{-7}}\right)^{-4}$

14. $\left(\dfrac{3^{-5}}{3^2}\right)^3 \times \left(\dfrac{3^5}{3^{-3}}\right)^{-2}$

15. $\left(\dfrac{9^{-5}}{9^{-2}}\right)^3 \times \left(\dfrac{9^{-4}}{9^6}\right)^3$

16. $\left(\dfrac{2^{-5}}{2^7}\right)^{-3} \times \left(\dfrac{2^{-8}}{2^{-5}}\right)^0$

17. $\left(\dfrac{4^6}{4^{-2}}\right)^3 \times \left(\dfrac{4^{-3}}{4^{-15}}\right)^4$

18. $4^{12} \times 4^{-3} \times 4^{-8} \times 4^{-8}$

19. $\dfrac{6^{-2}}{6^4} \times \dfrac{6^{-1}}{6^{-5}} \times \dfrac{6^8}{6^{-2}}$

20. $\left(\dfrac{2^{-6}}{2^{-2}} \times \dfrac{2^{-9}}{2^{-12}}\right)^{-3}$

EXERCISE 6H

1. Evaluate the following ***without*** using a calculator.

Examples

1. 5^3
$= 5 \times 5 \times 5$
$= \mathbf{125}$

2. 4^{-2}
$= \dfrac{1}{4^2}$
$= \mathbf{\dfrac{1}{16}}$

(a) 2^2 (b) 2^3 (c) 2^{-4} (d) 2^5 (e) 2^6
(f) 2^{-7} (g) 2^8 (h) 2^{-9} (i) 2^{10} (j) 2^{11}
(k) 3^2 (l) 3^3 (m) 3^{-4} (n) 5^2 (o) 5^3
(p) 10^2 (q) 10^{-3} (r) 10^4 (s) 10^{-5} (t) 10^6
(u) 8^3 (v) 9^{-2} (w) 6^{-3} (x) 4^{-4} (y) 7^4

2. Use the Index Laws to simplify the following then evaluate the answer ***without*** using a calculator.

Examples

1. $2^3 \times 2^4$
$= 2^7$
$= \mathbf{128}$

2. $2^5 \times 2^{-8}$
$= 2^{-3}$
$= \dfrac{1}{2^3}$
$= \mathbf{\dfrac{1}{8}}$

(a) $10^{-8} \times 10^5$ (b) $8^3 \times 8^0$ (c) $3^{-3} \times 3^3$

(d) $\dfrac{9^{13}}{9^{11}}$ (e) $\dfrac{(7^4)^{-5}}{(7^3)^{-6}}$ (f) $\dfrac{(6^7 \times 6^{-4})^2}{(6^{-5} \times 6^7)^2}$

(g) $\dfrac{2^{13}}{2^{11}} \times \dfrac{3^{-14}}{3^{-15}}$ (h) $\dfrac{5^{14}}{5^{12}} \times \dfrac{2^{-10}}{2^{-11}}$ (i) $\dfrac{(2^7 \times 2^{-8})^{-4}}{(2^{-5} \times 2^8)^{-2}}$

EXERCISE 6I

1. Use a calculator to evaluate the following problems.

(a) 15^4

(b) $13^5 - 21^4$

(c) $\dfrac{25^4 - 40^3}{5^3}$

(d) $(63^2 - 54^2)^2$

(e) $\left(\dfrac{72^3}{3^6}\right)^2$

(f) $\dfrac{93^2 - 15^2}{26}$

(g) $\dfrac{85^3 - 13^5}{2^4} - 23^3$

(h) $\left(\dfrac{474 - 118}{89}\right)^6$

(i) $\dfrac{2^{13}}{8^4} + \dfrac{3^{13}}{27^4}$

(j) $(5^2)^4 - (3^3)^3$

(k) $\dfrac{(7^6)^7}{(7^4)^9}$

(l) $(9^4)^2 - (17^2)^3 - (4^3)^4$

2. Use a calculator to evaluate the following problems.
Give answers as decimals correct to two decimal places.

(a) 0.41^{-3}

(b) $1.4^4 + 0.32^{-3}$

(c) $0.3^{-4} - 0.12^{-3}$

(d) $0.4^{-3} + 2.4^3$

(e) $\dfrac{5.3^{-4}}{3.6^{-6}}$

(f) $\dfrac{(2.3 \times 6.7)^{-2}}{5.18^{-4}}$

(g) $\dfrac{3.2^{-3} + 1.7^{-1}}{5.8^{-2} + 6.2^{-2}}$

(h) $\dfrac{(4.2^3 + 5.6^2)^{-2}}{(7.8^2 - 5.2^2)^{-3}}$

(i) $\left(\dfrac{628 - 5^4}{6^5 - 7780}\right)^{-3}$

Index Laws and Variables

The index laws also apply to variables as well as numbers.

Index Law 1

$a^m \times a^n = a^{m+n}$

EXERCISE 6J

Examples

1. $x^4 \times x^3$
$= x^{(4+3)}$
$= \boldsymbol{x^7}$

2. $m^6 \times m$
$= m^6 \times m^1$
$= m^{(6+1)}$
$= \boldsymbol{m^7}$

3. $n^5 \times n \times n^7$
$= n^5 \times n^1 \times n^7$
$= n^{(5+1+7)}$
$= \boldsymbol{n^{13}}$

4. $3a^3b^5 \times 2ab^4$
$= 3 \times a^3 \times b^5 \times 2 \times a \times b^4$
$= 3 \times 2 \times a^3 \times a \times b^5 \times b^4$
$= \boldsymbol{6a^4b^9}$

5. $5x^3y^2 \times x^5y^2 \times 4xy^3$
$= 5 \times x^3 \times y^2 \times x^5 \times y^2 \times 4 \times x \times y^3$
$= 5 \times 4 \times x^3 \times x^5 \times x \times y^2 \times y^2 \times y^3$
$= \boldsymbol{20x^9y^7}$

1. Simplify the following.

(a) $a^5 \times a^3$ (b) $n^4 \times n^4$ (c) $y^7 \times y^4$
(d) $b^6 \times b$ (e) $m^3 \times m^2 \times m^4$ (f) $x^4 \times x^5 \times x$
(g) $a \times a^6 \times a^7$ (h) $x^{10} \times x^5 \times x$ (i) $y^2 \times y \times y^5 \times y$

2. Simplify the following.

(a) $3a^5 \times 6a^3$
(b) $2x^4 \times 4x^7$
(c) $2m^3 \times 3m \times m^6$
(d) $3n^6 \times 2n^5 \times 2n^7$
(e) $2a^2b^2 \times a^3b^5$
(f) $xy \times 6x^3y^6$
(g) $4a^5y^6 \times 3a^2y$
(h) $2a^3b^4c \times 7a^4b^5c$
(i) $3m^5n^2 \times 2mn \times 2m^2n^5$
(j) $6x^5y^3z^4 \times 2x^3yz^2 \times xy^2z^2$
(k) $3a^4b^3c \times 8a^6bc^7$
(l) $5m^5n^3 \times 4m^6n^4 \times 2n \times 3m^6$

Index Law 2

$$a^m \div a^n = a^{m-n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

EXERCISE 6K

Examples

1. $x^6 \div x^4$
$= x^{6-4}$
$= \boldsymbol{x^2}$

2. $\dfrac{m^7}{m^4}$
$= m^{7-4}$
$= \boldsymbol{m^3}$

3. $8x^6y^7 \div 4x^5y$
$= \dfrac{8}{4}x^{6-5}y^{7-1}$
$= \boldsymbol{2xy^6}$

4. $\dfrac{18m^8n^6}{12m^3n^4}$
$= \dfrac{\overset{3}{\cancel{18}}}{\underset{2}{\cancel{12}}} m^{8-3}n^{6-4}$
$= \boldsymbol{\dfrac{3}{2}m^5n^2}$

1. Simplify the following.

(a) $x^5 \div x^2$ (b) $a^8 \div a^5$ (c) $n^9 \div n^3$ (d) $m^{10} \div m^3$

(e) $b^{11} \div b^8$ (f) $y^6 \div y$ (g) $p^7 \div p$ (h) $w^4 \div w^3$

(i) $\dfrac{m^{12}}{m^5}$ (j) $\dfrac{x^{13}}{x^4}$ (k) $\dfrac{n^{15}}{n}$ (l) $\dfrac{m^{11}}{m^{10}}$

2. Simplify the following.

(a) $10x^6y^7 \div 5x^3y^2$ (b) $16m^8n^9 \div 4m^5n^3$ (c) $24a^7b^5 \div 8a^4b^2$

(d) $16m^7n^6 \div 12m^5n^3$ (e) $8x^{10}y^8 \div 12x^7y^5$ (f) $20a^{11}b^7 \div 15a^8b^4$

(g) $12x^6y^6 \div 8x^4y$ (h) $20m^8n^{11} \div 24mn^{10}$ (i) $30a^8b^{12} \div 24a^5b$

(j) $\dfrac{18x^8y^4}{9x^5y^3}$ (k) $\dfrac{24m^8n^{12}}{20mn}$ (l) $\dfrac{24a^{10}b^{10}}{28a^9b}$

(m) $\dfrac{36m^8n^{11}}{12m^6n^5}$ (n) $\dfrac{32x^{15}y^{10}}{48x^9y^9}$ (o) $\dfrac{42m^7n^9}{14m^6n^8}$

(p) $\dfrac{54h^7i^{14}}{81h^6i^9}$ (q) $\dfrac{27a^7b^5}{54ab^5}$ (r) $\dfrac{48a^8b^7c^6}{36a^7b^7c^5}$

Index Law 3

$a^0 = 1$

EXERCISE 6L

Examples

1. $4^0 = \mathbf{1}$

2. $(a^2b^3)^0 = \mathbf{1}$

3. $3x^0 = 3 \times x^0 = 3 \times 1 = \mathbf{3}$

1. Simplify the following.

(a) y^0 (b) 5^0 (c) p^0 (d) 68^0

(e) $(5m)^0$ (f) $(3x)^0$ (g) $(7ab)^0$ (h) $(6xyz)^0$

2. Simplify the following.

(a) $5m^0$ (b) $3a^0$ (c) $4(3c^3)^0$ (d) $2a(3b^2)^0$

(e) $3^0 + 8^0$ (f) $2x^0 + (4x)^0$ (g) $(3m^0)^2$ (h) $5(n^5)^0$

(i) $4x^0 - (2x)^0$ (j) $5x^2y^0 + 2(xy^0)^2$ (k) $4x^0y^0z^0$ (l) $(3c^0)^2 - 2c^0$

3. Simplify the following.

(a) $\dfrac{m^6}{m^6}$ (b) $\dfrac{x^2z^5}{x^2z^3}$

(c) $\dfrac{6m^5n^4}{3m^2n^4}$ (d) $\dfrac{8a^4b^7}{10a^4b}$

(e) $\dfrac{12u^3v^3}{6u^3v^3}$ (f) $\dfrac{8m^8n^4}{12m^7n^4}$

(g) $\dfrac{18x^4y^3z^5}{24x^4yz^4}$ (h) $\dfrac{20a^5b^4c}{10a^4b^3c}$

Example

$$\frac{4m^5n^3}{3m^3n^3} = \frac{4}{3}m^{5-3}n^{3-3} = \frac{4}{3}m^2n^0 = \frac{4}{3}m^2 \times 1 = \mathbf{\frac{4}{3}m^2}$$

Index Law 4

$(a^m)^n = a^{m \times n}$

EXERCISE 6M

Examples

1. $(x^3)^4$
$= x^{3\times4}$
$= \boldsymbol{x^{12}}$

2. $(2m^2)^3$
$= 2^3 m^{2\times3}$
$= \boldsymbol{8m^6}$

1. Simplify the following.

(a) $(m^2)^6$ (b) $(x^3)^5$ (c) $(a^4)^6$ (d) $(c^6)^3$
(e) $(p^5)^6$ (f) $(b^7)^3$ (g) $(n^9)^4$ (h) $(y^4)^5$

2. Expand the brackets in the following.

(a) $(3x^2)^2$ (b) $(2m^3)^4$ (c) $(3m^5)^3$
(d) $(4n^6)^2$ (e) $(-6c^7)^2$ (f) $(-2n^4)^5$
(g) $(-3a^4)^3$ (h) $-(3u^6)^4$ (i) $(6a^3)^3$
(j) $-(5y^5)^3$ (k) $-(8c^4)^2$ (l) $-(-4y^3)^5$

Index Law 5

$(a \times b)^n = (ab)^n = a^n b^n$

$(a \div b)^n = \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Examples

1. $(x^3 \times y^5)^4$
$= (x^3 y^5)^4$
$= (x^3)^4 (y^5)^4$
$= \boldsymbol{x^{12} y^{20}}$

2. $(2m^2 n^5)^3$
$= 2^3 (m^2)^3 (n^5)^3$
$= \boldsymbol{8m^6 n^{15}}$

3. $(a^3 \div b^6)^5$
$= \left(\frac{a^3}{b^6}\right)^5$
$= \frac{(a^3)^5}{(b^6)^5}$
$= \boldsymbol{\frac{a^{15}}{b^{30}}}$

4. $\left(\frac{3m^2 n^3}{5x^3 y^5}\right)^2$
$= \frac{3^2 (m^2)^2 (n^3)^2}{5^2 (x^3)^2 (y^5)^2}$
$= \boldsymbol{\frac{9m^4 n^6}{25x^6 y^{10}}}$

EXERCISE 6N

1. Expand the brackets and simplify the following.

(a) $(a^2 \times b^3)^4$ (b) $(x^4 \times y^7)^2$ (c) $(p^2 \times q^9)^4$
(d) $(u^5 v^3)^6$ (e) $(m^5 n^8 o^6)^5$ (f) $(a^6 b^3 c^9)^3$
(g) $(3x^3 y^5)^2$ (h) $(4a^3 b)^2$ (i) $(-5mn^6)^3$
(j) $(8a^5 b^3)^2$ (k) $(-2a^4 bc^3)^6$ (l) $(9p^4 q^7 r^3)^2$
(m) $-(2m^3 n^6)^3$ (n) $-(3x^4 y^5)^4$ (o) $-(5u^6 v^7)^3$
(p) $-(-a^3 b^4)^3$ (q) $-(-4m^6 n^3)^4$ (r) $-(-2a^3 b^2)^7$
(s) $(6x^4 yz^5)^3$ (t) $(7a^4 b^3 c)^4$ (u) $(5p^3 q^7 r^8)^5$

2. Expand the brackets and simplify the following.

(a) $(a^5 \div b^2)^3$ (b) $(m^7 \div n^3)^2$ (c) $(x^7 \div y^5)^3$
(d) $\left(\frac{m^5}{n^4}\right)^2$ (e) $\left(\frac{a^6}{b^7}\right)^3$ (f) $\left(\frac{e^4}{r^2}\right)^5$
(g) $\left(\frac{m^3 n^4}{p^4 q^3}\right)^4$ (h) $\left(\frac{a^6 b^3}{x^2 y^8}\right)^2$ (i) $\left(\frac{m^5 p^3}{n^4 q^2}\right)^6$
(j) $\left(\frac{2a^3 b^4}{m^2 n^5}\right)^3$ (k) $\left(\frac{x^4}{3y^3 z^5}\right)^3$ (l) $\left(\frac{4m^2 n^5}{p^5 q^4}\right)^2$
(m) $\left(\frac{4m^3 n^6}{3a^2 b}\right)^3$ (n) $\left(\frac{5x^3 y^4}{6z^7}\right)^2$ (o) $\left(\frac{3a^4 b^9}{7x^5 y^7}\right)^2$
(p) $\left(\frac{2abc^3}{3x^2 y^3 z^4}\right)^3$ (q) $\left(\frac{5}{m^2 n^3 p^2}\right)^4$ (r) $\left(\frac{7mn^7}{10p^4}\right)^3$

Index Law 6

$$a^{-m} = \frac{1}{a^m}$$

EXERCISE 6O

Examples

1. $x^{-5} = \frac{1}{x^5}$

2. $5n^{-6} = \frac{5}{n^6}$

3. $2m^4n^{-3} = \frac{2m^4}{n^3}$

4. $\frac{2}{3}x^2y^{-4} = \frac{2x^2}{3y^4}$

5. $\frac{1}{6}a^3b^{-5}c^{-2} = \frac{a^3}{6b^5c^2}$

1. Write the following with positive indices.

(a) x^{-8} (b) 3^{-4} (c) a^{-2} (d) e^{-10} (e) c^{-1}

(f) $3m^{-4}$ (g) xy^{-2} (h) $4mn^{-7}$ (i) $6x^2y^{-3}$ (j) $8a^{-2}b^9$

2. Write the following with positive indices.

(a) $3m^2n^{-6}$ (b) $7a^3b^{-5}$ (c) $a^4b^3c^{-6}$ (d) $6x^{-5}y^3$

(e) $\frac{3}{4}m^2n^{-3}$ (f) $\frac{5}{7}a^3b^{-4}$ (g) $\frac{1}{2}x^{-4}y^3$ (h) $\frac{1}{3}x^3yz^{-2}$

(i) $\frac{4}{9}a^5b^{-3}c$ (j) $\frac{2}{7}p^4q^{-3}r^{-2}$ (k) $\frac{3}{8}a^{-2}b^{-3}c^{-2}$ (l) $\frac{1}{5}v^{-3}w^2x^{-7}$

Summary of Index Laws

Index Law 1

$$a^m \times a^n = a^{m+n}$$

Index Law 2

$$a^m \div a^n = a^{m-n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

Index Law 3

$$a^0 = 1$$

Index Law 4

$$(a^m)^n = a^{m \times n}$$

Index Law 5

$$(a \times b)^n = (ab)^n = a^n b^n$$

$$(a \div b)^n = \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Index Law 6

$$a^{-m} = \frac{1}{a^m}$$

Using All The Index Laws

EXERCISE 6P

1. Simplify the following.

Examples

1. $\dfrac{x^3 \times x^6}{x^4}$

$= \dfrac{x^9}{x^4}$

$= \boldsymbol{x^5}$

2. $\dfrac{4m \times 6m^5}{12m^2}$

$= \dfrac{24m^6}{12m^2}$

$= \boldsymbol{2m^4}$

3. $\dfrac{(2x^3)^3}{6x^4}$

$= \dfrac{2^3x^9}{6x^4}$

$= \dfrac{8x^9}{6x^4}$

$= \boldsymbol{\dfrac{4x^5}{3}}$

(a) $\dfrac{x^5 \times x^6}{x^3}$ (b) $\dfrac{m^6 \times m^2}{m}$ (c) $\dfrac{c^7 \times c}{c^5}$

(d) $\dfrac{a^2b^3 \times a^6b^4}{a^3b^2}$ (e) $\dfrac{m^5n^3 \times m^4n^6}{m^4n}$ (f) $\dfrac{6x^2 \times 2x^5}{4x^3}$

(g) $\dfrac{5m^4 \times 4m^3}{10m^5}$ (h) $\dfrac{3x^7 \times 6x^6}{2x^4}$ (i) $\dfrac{8c^6 \times 3c}{16c^4}$

(j) $\dfrac{b^3 \times 2b^4}{8b^5}$ (k) $\dfrac{4x \times 5x^3}{12x^2}$ (l) $\dfrac{9n^2 \times 4n^4}{6n^6}$

(m) $\dfrac{(4x^3)^2}{2x^3}$ (n) $\dfrac{(3m^4)^2}{6m^5}$ (o) $\dfrac{(2x^4)^4}{10x}$

2. Simplify the following.

Examples

1. $\dfrac{(x^2y^3)^2 \times (x^3y)^3}{(x^3y^2)^2 \times x^4y^5}$

$= \dfrac{x^4y^6 \times x^9y^3}{x^6y^4 \times x^4y^5}$

$= \dfrac{x^{13}y^9}{x^{10}y^9}$

$= x^3y^0$

$= \boldsymbol{x^3}$

2. $\dfrac{3(x^4y^3)^3}{(2x^2y)^3} \times \dfrac{4(x^5y^2)^4}{(x^3y^2)^2}$

$= \dfrac{3x^{12}y^9}{8x^6y^3} \times \dfrac{4x^{20}y^8}{x^6y^4}$

$= \dfrac{12x^{32}y^{17}}{8x^{12}y^7}$

$= \dfrac{\mathbf{3}}{\mathbf{2}}\, x^{20}y^{10}$

(a) $\dfrac{(x^4y^2)^3 \times (xy^5)^3}{(x^2y)^4 \times x^2y^7}$

(b) $\dfrac{m^5n^8 \times (m^4n^2)^{-3}}{(m^2n^4)^2 \times (mn^3)^2}$

(c) $\dfrac{(a^5b^3)^4 \times (a^3b^2)^3}{(a^2b^3)^2 \times (a^4b)^{-3}}$

(d) $\dfrac{3(x^4y^3)^2 \times (4x^3y^6)^2}{2(xy^2)^2 \times 6x^2y^5}$

(e) $\dfrac{(3m^5n^3)^2 \times 8(m^4n^2)^3}{2(m^3n^2)^2 \times (3m^5n^2)^3}$

(f) $\dfrac{(3x^5y^{-3})^3 \times (4x^3y^6)^2}{2(x^6y^2)^2 \times (6x^2y^{-5})^2}$

(g) $\dfrac{(x^2y^3)^{-4}}{(3xy^2)^2} \times \dfrac{6(x^3y^6)^3}{(2x^3y^5)^2}$

(h) $\dfrac{(3m^6)^2}{(2m^3)^3} \times \dfrac{4(n^5)^2}{3(mn^2)^3}$

(i) $\dfrac{6a^3b^5}{(2a^2b^3)^4} \times \dfrac{(4a^5b^6)^2}{(3a^2b)^2}$

(j) $\dfrac{(10x^3)^2}{(4y^2)^3} \times \dfrac{(8y^5)^2}{(5x)^2}$

(k) $\dfrac{4(m^3n^2)^4}{(3m^2n)^3} \times \dfrac{(9m^3n^6)^2}{(2mn^3)^5}$

(l) $\dfrac{(2a^3b^4)^3}{4(a^5b^{10})^{-3}} \times \dfrac{(3a^2b^5)^4}{(9ab)^2}$

3. Simplify the following.

Examples

1. $\dfrac{3a^2}{5} \div \dfrac{9a}{10}$

$= \dfrac{3a^2}{5} \times \dfrac{10}{9a}$

$= \dfrac{30a^2}{45a}$

$= \mathbf{\dfrac{2a}{3}}$

2. $\dfrac{(2mn^4)^2}{(3m^2n^3)^2} \div \dfrac{(6m^2n^3)^2}{(3m^2n^4)^3}$

$= \dfrac{4m^2n^8}{9m^4n^6} \div \dfrac{36m^4n^6}{27m^6n^{12}}$

$= \dfrac{4m^2n^8}{9m^4n^6} \times \dfrac{27m^6n^{12}}{36m^4n^6}$

$= \dfrac{1\cancel{4}m^2n^8}{\cancel{9}_1m^4n^6} \times \dfrac{1\cancel{27}m^6n^{12}}{\cancel{36}_3m^4n^6}$

$= \dfrac{1m^8n^{20}}{3m^8n^{12}}$

$= \mathbf{\dfrac{n^8}{3}}$

(a) $\dfrac{4x^4}{3} \div \dfrac{28x}{15}$

(b) $\dfrac{15c^7}{8} \div \dfrac{5c^3}{16}$

(c) $\dfrac{20}{9a^3} \div \dfrac{15}{18a^6}$

(d) $\dfrac{12a^5}{9b^3} \div \dfrac{4}{18b^3}$

(e) $\dfrac{7x^7}{8y^3} \div \dfrac{21x}{4y^5}$

(f) $\dfrac{30x^4}{25y^3} \div \dfrac{6x^4}{5y^3}$

(g) $\dfrac{6x^3y^5}{(5x^2y^4)^2} \div \dfrac{3x^8y^5}{25(x^5y^5)^2}$

(h) $\dfrac{(2x^2y^3)^5}{(3x^2y^4)^2} \div \dfrac{(4xy^2)^3}{(6x^3y^6)^2}$

(i) $\dfrac{(8a^3b^7)^3}{(5a^4b^5)^3} \div \dfrac{(2a^2b^3)^9}{(10a^8b^7)^3}$

(j) $\dfrac{(3m^2n^3)^4}{(2mn^3)^6} \div \dfrac{(3m^2n^3)^3}{(4m^5n^{10})^2}$

Scientific Notation (Standard Form)

Scientific notation, or ***standard form***, is a convenient way of writing very large or very small numbers. It is often used in science because of the large numbers (for example the distance between planets) or small numbers (for example the mass of an atom). Scientific notation involves moving the decimal point to after the first non-zero digit and using powers of 10.

EXERCISE 6Q

1. Convert the following numbers into scientific notation.

Move the decimal point to after the first digit.

Examples

1. 6000
$= 6 \times 1000$
$= \mathbf{6 \times 10^3}$

2. 935 000
$= 9.35 \times 100\,000$
$= \mathbf{9.35 \times 10^5}$

(a) 9000 (b) 50 000 (c) 700 000 000
(d) 275 000 (e) 320 000 (f) 82 100 000
(g) 365 470 000 (h) 230 000 000 000 (i) 50 001
(j) 6587 (k) 254.34 (l) 1537.8

2. Convert the following numbers into scientific notation.

Move the decimal point to after the first digit.

Examples

1. 0.0076
$= 7.6 \div 1000$
$= 7\,6 \times \frac{1}{1000}$
$= \mathbf{7.6 \times 10^{-3}}$

2. 0.000 000 582
$= 5.82 \div 10\,000\,000$
$= 5.82 \times \frac{1}{10\,000\,000}$
$= \mathbf{5.82 \times 10^{-7}}$

(a) 0.0005 (b) 0.000 009 (c) 0.000 000 004
(d) 0.000 13 (e) 0.006 92 (f) 0.000 057 9
(g) 0.002 045 (h) 0.000 006 (i) 0.000 000 068 1
(j) 0.000 020 204 (k) 0.000 000 000 2 (l) 0.000 000 005 7

3. Convert the following numbers into numeral form.

(a) 3.6×10^{4} (b) 8×10^{6} (c) 6.71×10^{5}
(d) 2.455×10^{8} (e) 7.201×10^{7} (f) 3.82×10^{11}
(g) 2.7×10^{-3} (h) 7×10^{-5} (i) 9.3×10^{-4}
(j) 4.83×10^{-7} (k) 5.917×10^{-9} (l) 1.008×10^{-10}

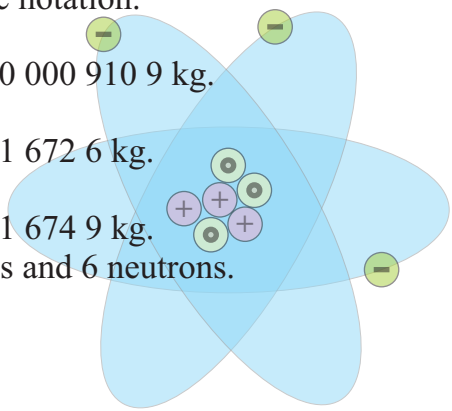
4. Atoms consist of three particles - electrons, protons and neutrons.
Write the following dimensions in scientific notation.

(a) The mass of an electron is
0.000 000 000 000 000 000 000 000 000 000 910 9 kg.

(b) The mass of a proton is
0.000 000 000 000 000 000 000 000 001 672 6 kg.

(c) The mass of a neutron is
0.000 000 000 000 000 000 000 000 001 674 9 kg.

(d) A carbon atom has 6 electrons, 6 protons and 6 neutrons.
Find the mass of a carbon atom.



5. Write the following quantities in scientific notation.

(a) The mass of the Sun is:
1 990 000 000 000 000 000 000 000 000 000 kg.

(b) The temperature at the core of the Sun is:
15 000 000°C.

6. (a) Write the following planet masses in scientific notation.

Mercury - 330 000 000 000 000 000 000 000 kg
Venus - 4 870 000 000 000 000 000 000 000 kg
Earth - 5 980 000 000 000 000 000 000 000 kg
Mars - 642 000 000 000 000 000 000 000 kg
Jupiter - 1 900 000 000 000 000 000 000 000 000 kg
Saturn - 569 000 000 000 000 000 000 000 000 kg
Uranus - 86 900 000 000 000 000 000 000 000 kg
Neptune - 102 000 000 000 000 000 000 000 000 kg
Pluto - 12 000 000 000 000 000 000 000 kg

(b) List the planets in order from the lightest to the heaviest.
Pluto is now classified as a dwarf planet.

7. There are 400 billion stars in the milky way galaxy.
Write this in scientific notation (one billion in this case is one thousand million).

8. Convert the following measurements to scientific notation.
(a) The width of the smallest atom (hydrogen) is 0.000 000 000 1 m.
(b) The width of a human egg is 0.000 12 m.
(c) The width of an E. coli bacterium is 0.000 002 m.
(d) The width of a human red blood cell is 0.000 007 m
(e) The width of a human hair is 0.0001 m.

9. (a) The speed of light is approximately 3×10^8 metres per sec (m/s). Convert this speed to kilometres per second (km/s).

(b) The distance from the Earth to the Sun is 150 million kilometres. Convert this distance into scientific notation.

(c) By using the formula for time taken shown here calculate the time it takes for light to travel from the Sun to the Earth.

$$\textit{time taken} = \frac{\textit{distance}}{\textit{speed}}$$

Give answer in:

(i) seconds (ii) minutes and seconds

(d) The distance from the Earth to the Moon is 384 400 km. Find the time it takes light to travel from the Earth to the Moon. Give answer in seconds correct to two decimal places.

10. Communications satellites are used to relay and amplify telecommunications signals around the Earth. If a satellite is in a geosynchronous orbit it is 36 000 km above the Earth.

Telecommunications signals travel at the speed of light.

How long would it take a signal to travel from the Earth to the satellite and back to the Earth?

11. Another way for telecommunications signals to being transmitted is through optic fibres or via telecommunications relays. These signals also travel at the speed of light.

Find the time it would take a signal to travel the following distances. Give answers in seconds in scientific notation.

(a) Sydney to Perth - 3300 km.

(b) Brisbane to Ballina - 210 km

(c) 900 m

(d) 60 cm

(e) 30 mm

12. A unit of time often used when dealing with very small durations of time is the nanosecond (ns).

1 nanosecond = 10^{-9} seconds

Convert the times in question 11 to nanoseconds.

PROBLEM SOLVING

1. The speed of light is 3×10^8 metres per second.
(a) How far would light travel in:
(i) 2 seconds (ii) 1 minute (iii) 1 hour
(b) A ***light year*** is the distance that light travels in a year.
Calculate how many kilometres are in one light year.
(c) The milky way galaxy is 100 000 light years across.
How many kilometres is this?
(d) Find the name of the nearest star to our solar system and how far it is from Earth in light years and kilometres.

2. Scientists were studying the growth of bacteria on food after it was taken out of a refrigerator.
They found that the number of bacteria on an item of food could be calculated by using the following formula:

$n = 200 \times 5^t$

where n = the number of bacteria
and t = the number of hours after the food was taken out of the refrigerator

Example Find the number of bacteria on the food after 2 hours:

$t = 2$ $\quad n = 200 \times 5^2$
$= 200 \times 25$
$\boldsymbol{n = 5000}$ **bacteria**

(a) Find the number of bacteria after: (i) 3 hours (ii) 5 hours
(b) After how many hours are there 7.8125×10^7 bacteria?

PUZZLE

Rearrange the letters in the following phrases to make words or phrases from this chapter.

1. INFECTION IS INTO CAT
2. AXLES WIND
3. TRANSFORM DAD
4. BARE BUSMEN
5. IDIOTIC EVE SPINS
6. CUBAN REALISM

CHAPTER REVIEW

1. Write $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ in index form.

2. Simplify the following.

(a) $3^5 \times 3^4$ (b) $\frac{6^8}{6^3}$ (c) $4^9 \div 4$ (d) $3^2 + 2^0$

(e) $(7^5)^4$ (f) $(5^3 \times 5^7)^2$ (g) $\left(\frac{9^6}{5^3}\right)^4$ (h) $(4^5 \div 7^8)^4$

3. Write the following with positive indices.

(a) 6^{-5} (b) 5×3^{-8} (c) $\frac{4}{8 \times 7^{-3}}$ (d) $\frac{2^9 \times 7^{-6}}{5^{-8} \times 4^3}$

4. Simplify the following.

(a) $x^5 \times x^4$
(b) $3m^6n^2 \times 4m^5n^4$
(c) $(a^3)^4$
(d) $(2m^3n^2)^5$
(e) $(-a^2b^3)^3$
(f) $-(3p^2q^3r^2)^4$
(g) $(3m)^0$
(h) $6a^0 + (2b)^0$

(i) $\frac{a^6 \times 6a^7}{2a^3}$

(j) $\frac{(3x^2y^5)^4}{(9x^3y^2)^2}$

(k) $\frac{(2m^3n^5)^4}{(3mn^3)^3} \times \frac{(9m^5n^6)^2}{(4m^9n^{11})^2}$

(l) $\frac{(5x^3y^5)^2}{(6x^4y^6)^2} \div \frac{(10x^4y^3)^2}{(4x^5y^4)^3}$

5. Write the following with positive indices.

(a) x^{-6} (b) $8m^2n^{-3}$ (c) $\frac{2}{5}x^5y^{-2}z^{-4}$

6. Convert the following numbers into scientific notation.

(a) 8 620 000 000 (b) 0.000 000 057 2

7. Convert the following numbers into numeral form.

(a) 6.71×10^4 (b) 3.01×10^{-6}

8. Use a calculator to evaluate the following.

(a) $4^6 - 6^4$ (b) $\left(\frac{8^6}{2^{17}}\right)^6$ (c) $\frac{2^{-10}}{8^{-4}} + \frac{3^{-12}}{9^{-6}}$