Binary Numbers

The ***decimal*** system of expressing numbers is based on powers of **10** - see the table below for examples of numbers expressed in decimal form.

		Powers of 10					
		10^5	10^4	10^3	10^2	10^1	10^0
		100 000	10 000	1000	100	10	1
Number	87					8	7
	1253			1	2	5	3
	34 801		3	4	8	0	1
	926 083	9	2	6	0	8	3

These numbers can be shown in their expanded form to show the power of 10 represented by each digit.

$$\begin{aligned} 87 &= (8 \times 10^1) + (7 \times 10^0) \\ &= (8 \times 10) + (7 \times 1) \\ &= 80 + 7 \end{aligned}$$

$$\begin{aligned} 1253 &= (1 \times 10^3) + (2 \times 10^2) + (5 \times 10^1) + (3 \times 10^0) \\ &= (1 \times 1000) + (2 \times 100) + (5 \times 10) + (3 \times 1) \\ &= 1000 + 200 + 50 + 3 \end{aligned}$$

$$\begin{aligned} 34\,801 &= (3 \times 10^4) + (4 \times 10^3) + (8 \times 10^2) + (0 \times 10^1) + (1 \times 10^0) \\ &= (3 \times 10\,000) + (4 \times 1000) + (8 \times 100) + (0 \times 10) + 1 \\ &= 30\,000 + 4000 + 800 + 1 \end{aligned}$$

EXERCISE 1

1. Express 926 083 in the expanded form showing the power of 10 represented by each digit. (See examples above)

2. Express the following decimal numbers in expanded form showing the power of 10 represented by each digit.

(a) 579 (b) 7008 (c) 12 730 (d) 481 597
(e) 690 348 (f) 2 451 093 (g) 51 672 901 (h) 818 403 592

The ***binary*** system of expressing numbers is based on powers of **2**. There are only two digits used in numbers expressed in binary form - **0** and **1**.
See the table below for examples of numbers expressed in binary form.

		Powers of 2					
		2^5	2^4	2^3	2^2	2^1	2^0
		32	16	8	4	2	1
Binary Number	11					1	1
	101				1	0	1
	1111			1	1	1	1
	10101		1	0	1	0	1
	110110	1	1	0	1	1	0

By expressing binary numbers in their expanded form the equivalent decimal numbers can be found.

$11 = (1 \times 2^1) + (1 \times 2^0)$
$= (1 \times 2) + (1 \times 1)$
$= 2 + 1$
$=$ **3** (decimal equivalent)

$101 = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
$= (1 \times 4) + (0 \times 2) + (1 \times 1)$
$= 4 + 0 + 1$
$=$ **5** (decimal equivalent)

$1111 = (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= (1 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1)$
$= 8 + 4 + 2 + 1$
$=$ **15** (decimal equivalent)

$10101 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
$= (1 \times 16) + (0 \times 8) + (1 \times 4) + (0 \times 2) + 1$
$= 16 + 0 + 4 + 0 + 1$
$=$ **21** (decimal equivalent)

EXERCISE 2

1. (a) Express 110110 in the expanded form showing the power of 2 represented by each digit. (See examples on previous page)
(b) What is the decimal equivalent of the binary number 110110?

Numbers with base 10 (decimal numbers) are represented with a subscript 10:
for example, 23_{10} 296_{10} 4904_{10}
Numbers with base 2 (binary numbers) are represented with a subscript 2:
for example, 101_2 11011_2 10101010_2

From the examples on the previous page:

$11_2 = 3_{10}$
$101_2 = 5_{10}$
$1111_2 = 15_{10}$
$10101_2 = 21_{10}$

2. From question 1 complete the following:

$110110_2 = \_\_\_\_\__{10}$

3. Find the following:

(a) 2^6 (b) 2^7 (c) 2^8 (d) 2^9 (e) 2^{10} (f) 2^{11}
(g) 2^{12} (h) 2^{13} (i) 2^{14} (j) 2^{15} (k) 2^{16} (l) 2^{17}

4. Convert the following binary numbers to numbers with a base 10.

(a) 111_2
(b) 1001_2
(c) 1011_2
(d) 110001_2
(e) 101111_2
(f) 110011_2
(g) 1101010_2
(h) 11001010_2
(i) 1010110101_2
(j) 100110011101_2
(k) 10100001111_2
(l) 10011111011010_2
(m) 10000000000000_2
(n) 1000000000000100_2
(o) 100001000010000_2
(p) 1111111111111111_2

5. Convert the following decimal numbers (base 10) to binary numbers (base 2).

To covert from a decimal number (base 10) to binary number (base 2) follow these steps.

Step 1 Find the largest power of 2 (2, 4, 8, 16, 32, 64 ....) that is smaller than or equal to the number.
Step 2 Subtract this from the number and find the remainder.
Step 3 Repeat step 1 for the remainder.
Step 4 Repeat these steps until there is a remainder of 0 or 1.
Step 5 List the number of each power of 2 starting with the largest to find the binary number.

Example 58_{10}

Step 1 The largest power of 2 less than or equal to 58 is 32.
Step 2 58 - 32 = 26
Step 3 The largest power of 2 less than or equal to 26 is 16.
Step 4 26 - 16 = 10
The largest power of 2 less than or equal to 10 is 8.
10 - 8 = 2
The largest power of 2 less than or equal to 2 is 2.
2 - 2 = 0
Step 5 $(\mathbf{1} \times 32) + (\mathbf{1} \times 16) + (\mathbf{1} \times 8) + (\mathbf{0} \times 4) + (\mathbf{1} \times 2) + (\mathbf{0} \times 1)$
$= 111010_2$

$\mathbf{58_{10} = 111010_2}$

(a) 13_{10}	(b) 17_{10}	(c) 20_{10}	(d) 22_{10}
(e) 24_{10}	(f) 31_{10}	(g) 32_{10}	(h) 39_{10}
(i) 45_{10}	(j) 60_{10}	(k) 73_{10}	(l) 92_{10}
(m) 111_{10}	(n) 144_{10}	(o) 225_{10}	(p) 257_{10}
(q) 437_{10}	(r) 789_{10}	(s) 1030_{10}	(t) 1047_{10}
(u) 3278_{10}	(v) 4311_{10}	(w) 6783_{10}	(x) 9460_{10}

EXERCISE 3

When adding binary numbers remember that

$$\mathbf{1_2 + 1_2 = 10_2}$$

and $$\mathbf{1_2 + 1_2 + 1_2 = 11_2}$$

Example $11101_2 + 1111_2$

$$\begin{array}{r} {}^{1}1{}^{1}1{}^{1}1{}^{1}01 \\ +\quad 1111 \\ \hline 101100 \\ \hline \end{array}$$

1 + 1 = 10
Write 0
Carry 1

$$\mathbf{11101_2 + 1111_2 = 101100_2}$$

1. (a) $100_2 + 11_2$
(b) $11_2 + 1_2$
(c) $101_2 + 101_2$
(d) $1011_2 + 11_2$
(e) $101011_2 + 11011_2$
(f) $1011_2 + 111011_2$
(g) $110101_2 + 1100111_2$
(h) $11111_2 + 11111_2$
(i) $10110111_2 + 111000_2$
(j) $10000001_2 + 111100001_2$
(k) $111000111_2 + 111100101_2$
(l) $111011111_2 + 111111110011_2$

The above answers can be checked by changing the binary numbers to decimal numbers and adding.

Using the above example

$$11101_2 + 1111_2 = 101100_2$$
$$29_{10} + 15_{10} = 44_{10}$$

2. Check the answers to the addition problems in question 1 by converting the binary numbers to decimal numbers and performing the addition.

EXERCISE 4

1. Perform the following multiplications of binary numbers.
2. Check the answers by converting the binary numbers to decimal numbers and performing the multiplications.

Example $111_2 \times 101_2$

$$\begin{array}{r} 111 \\ \times\ 101 \\ \hline 111 \\ 0000 \\ 11100 \\ \hline 100011 \\ \hline \end{array}$$

$111_2 \times 101_2$
$= \mathbf{100011_2}$

Converting to decimal numbers: $111_2 = 7_{10}$
$101_2 = 5_{10}$

$7_{10} \times 5_{10} = \mathbf{35_{10}\ (100011_2)}$

(a) $101_2 \times 11_2$
(b) $110_2 \times 11_2$
(c) $1010_2 \times 11_2$
(d) $1111_2 \times 10_2$
(e) $11001_2 \times 11_2$
(f) $101010_2 \times 11_2$
(g) $111111_2 \times 11_2$
(h) $11101111_2 \times 11_2$
(i) $11011_2 \times 101_2$
(j) $1110101_2 \times 101_2$
(k) $11010111_2 \times 101_2$
(l) $101111011_2 \times 101_2$
(m) $101011_2 \times 100_2$
(n) $110011001_2 \times 100_2$
(o) $1010111_2 \times 100_2$
(p) $11001111011_2 \times 100_2$

